

The motion of a single exciton in a bilayer quantum antiferromagnet

Louk Rademaker, Kai Wu



Universiteit Leiden
The Netherlands

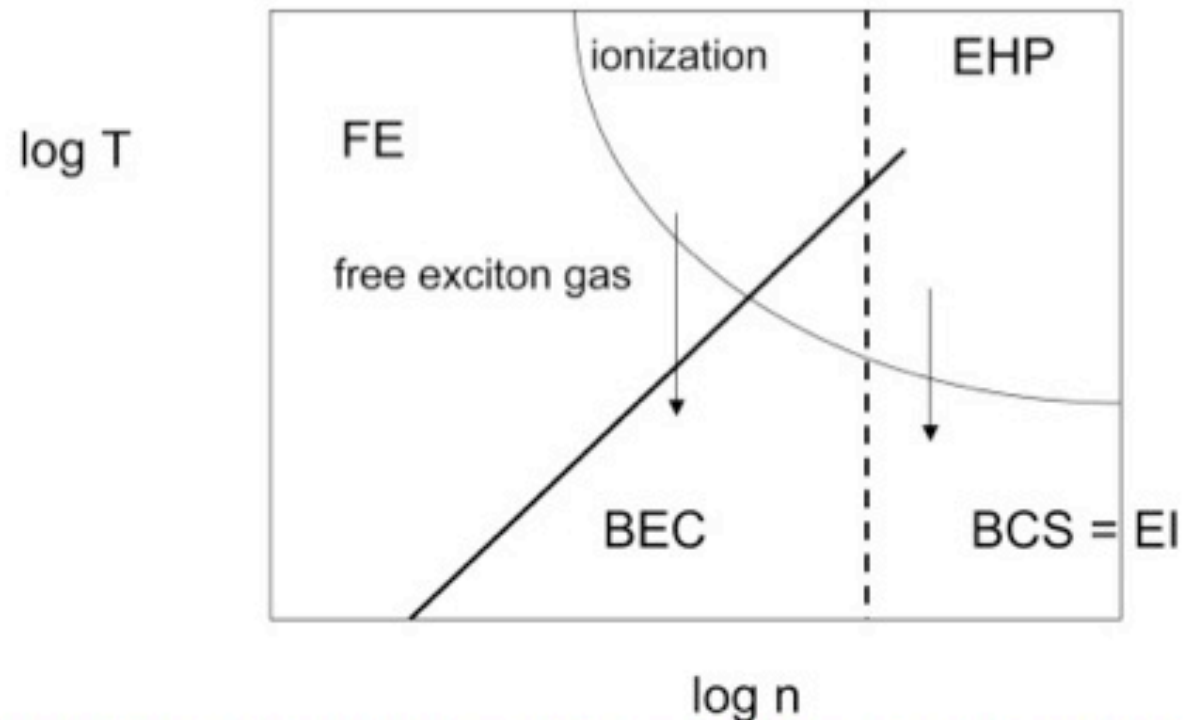
Overview

- Review: excitons and heterostructures
- Holes in strongly correlated layers
- Strongly correlated bilayers
- Excitons in strongly correlated bilayers



Excitons: a little theory

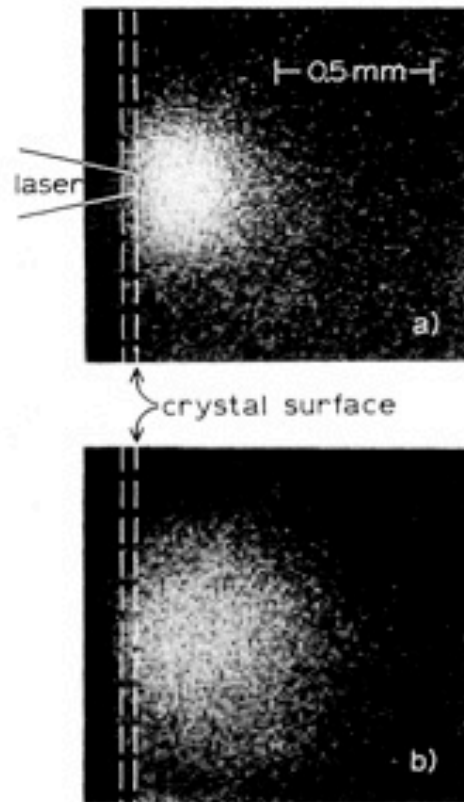
- Bound electron-hole pair
- 'Hydrogen-like' with Bohr radius: $a = \frac{\hbar^2 \epsilon}{e^2 \mu}$
- General phase diagram:



Ref: Snoke, cond-mat/1011.3844; Moskalenko & Snoke, "BEC of Excitons" (2000)

Excitons: experiments

- Bulk laser excited excitons in Cu_2O

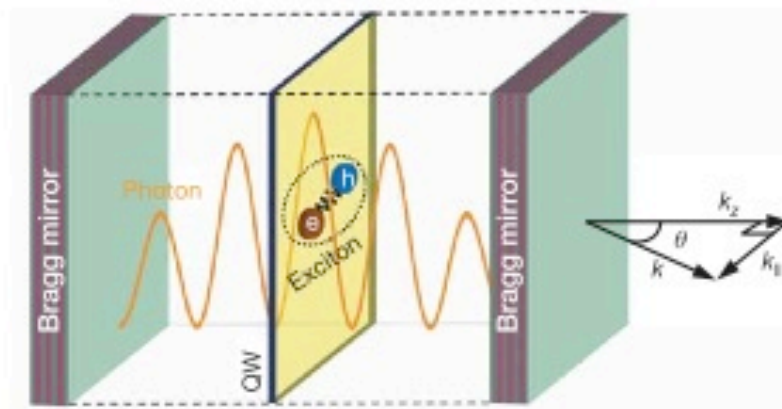


Spectral function > BEC?
Transport > Superfluidity?

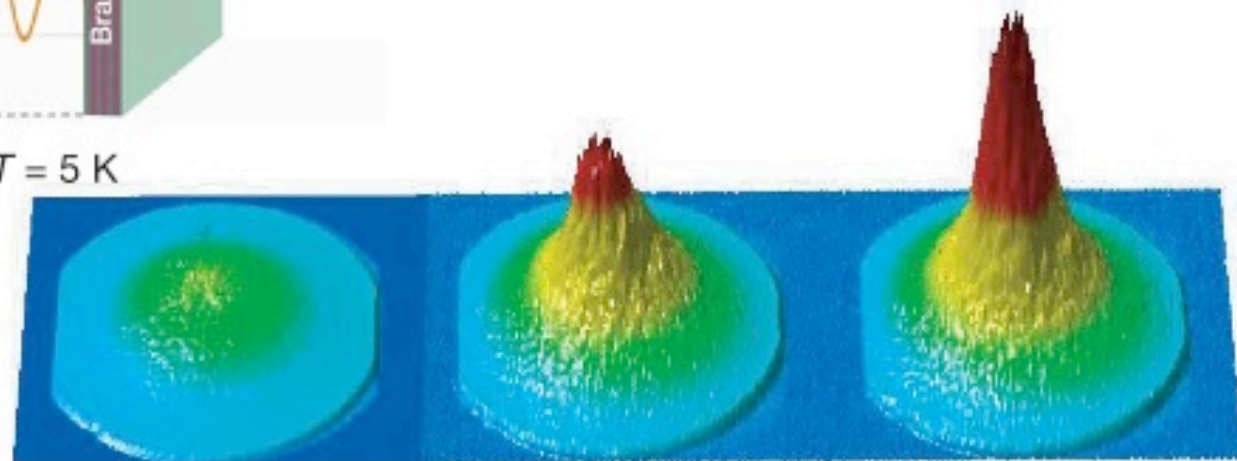
Ref: Trauernicht et al, PRL 52, 855 (1984)

Excitons: experiments

- Trapped exciton-polaritons
- BEC at 19 K, coherence proven



$T = 5 \text{ K}$



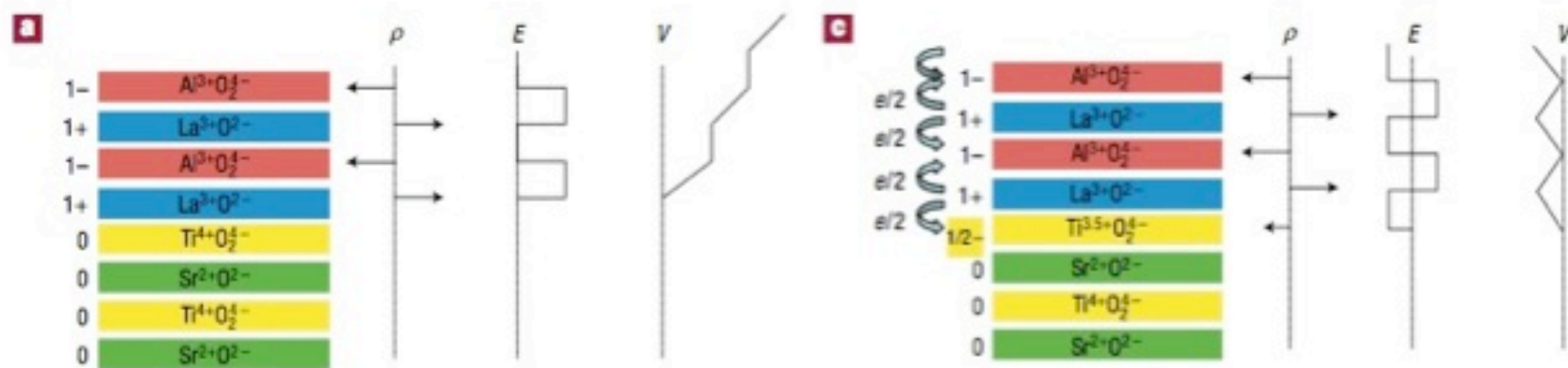
Ref: Kasprzak et al, Nature 443, 409 (2006)

Double layer excitons

- Double layer excitons
 - Features:
 - Tunneling, zero Hall, drag
 - Experimental realization:
- A 3D schematic diagram of a double layer exciton. It shows a central gray rectangular block labeled 'insulator' with a vertical double-headed arrow on its left side labeled 'd'. Above the insulator is a blue layer labeled 'hole layer' containing several circles with a plus sign (+). Below the insulator is a green layer labeled 'electron layer' containing several circles with a minus sign (-). A vertical orange cylinder, labeled 'exciton' at its base, passes through the center of the insulator layer, connecting the hole layer and the electron layer.
- ## Quantum Hall Bilayers
- A three-part diagram illustrating the formation of double layer excitons in Quantum Hall Bilayers. Part (a) shows two parallel horizontal planes, each containing a collection of small spheres (orange on top, blue on bottom). Part (b) shows a single plane with a grid pattern and a yellow arrow pointing upwards, labeled 'Magnetic field' and 'B'. Part (c) shows two parallel planes with grid patterns, one orange and one green, with a yellow arrow pointing upwards between them, labeled 'B'. A label 'Particle-hole transformation' with an arrow points from part (b) towards part (c).
- ## Coupled Quantum Wells
- A diagram of two coupled quantum wells. The wells are represented by two horizontal lines with a central dip. A red dot labeled 'e' is in the upper well, and a black dot labeled 'h' is in the lower well. A double-headed arrow connects the two dots, representing an exciton.
- Ref: Eisenstein & MacDonald, Nature 432, 691 (2004);
Butov et al, Nature 418, 751 (2002)
- Leiden University. The university to discover.

Detour: Oxide Heterostructures

- LAO/STO interface conductivity
- Electronic reconstruction



- Twente 2011: NCCO/LSCO interfaces

Ref: Ohtomo & Hwang, Nature 427, 423 (2004);
Nakagawa et al, Nat. Mater. 5, 204 (2006)

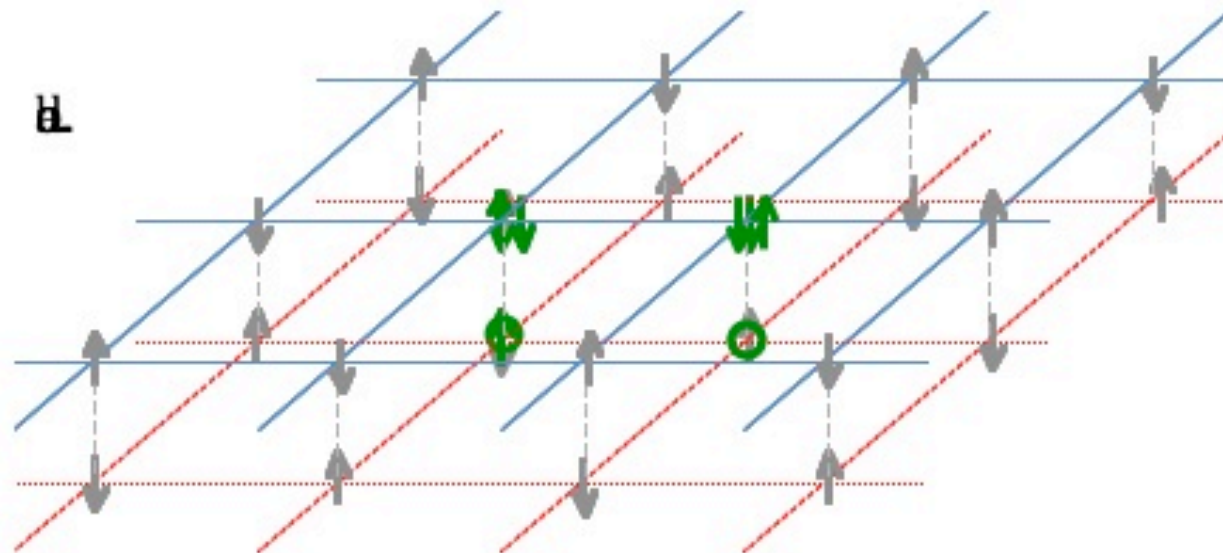
Our research

- What are the properties of excitons in electron-hole strongly correlated oxide heterostructures?
- Problem:
No (good) theory for the doped Mott insulator



The exciton t-J model

- Assumptions:
 - only interface layers are relevant
 - excitons are bound states



Effective t-J model of Excitons

$$H = H_t + H_J$$

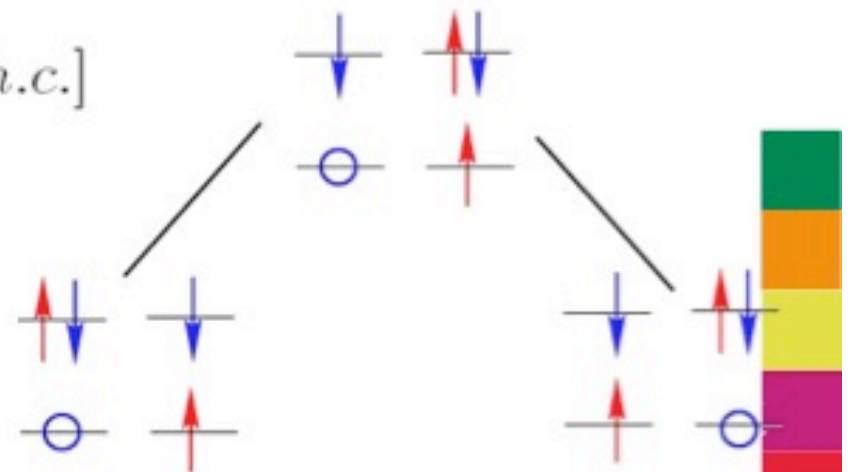
$$H_t = -t \sum_{\langle ij \rangle, \sigma} (c_{i1\sigma}^\dagger c_{j1\sigma} + c_{i2\sigma}^\dagger c_{j2\sigma} + h.c.) - V \sum_i n_{i1}^d n_{i2}^h$$

$$H_J = J_1 \sum_{\langle ij \rangle} (\vec{s}_{i1} \cdot \vec{s}_{j1} + \vec{s}_{i2} \cdot \vec{s}_{j2}) + J_2 \sum_i (\vec{s}_{i1} \cdot \vec{s}_{i2})$$

Second-order perturbation

$$H_t = -\frac{2t^2}{V} \sum_{\langle ij \rangle \sigma \sigma'} (c_{i1\sigma}^\dagger c_{j1\sigma} c_{i2\sigma'}^\dagger c_{j2\sigma'} + h.c.)$$

$$= -t_e \sum_{\langle ij \rangle \sigma \sigma'} [(c_{i1\sigma}^\dagger c_{i2\sigma'}) (c_{j2\sigma'}^\dagger c_{j1\sigma}) + h.c.]$$



How does a single exciton move in the bilayer antiferromagnet?

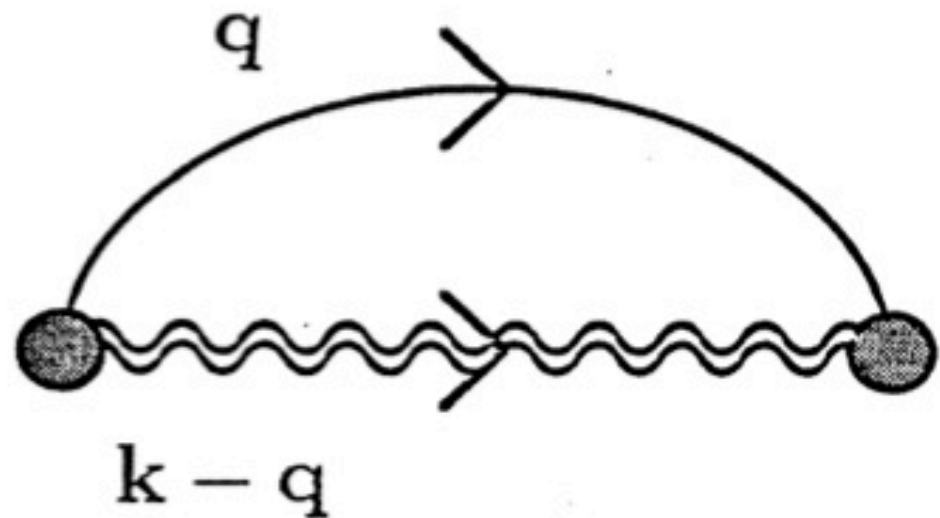
- A single hole moves in a quantum antiferromagnet
 - Effective Hamiltonian
 - Self-consistent Born Approximation;
- Effective Hamiltonian for the Exciton from the bilayer Heisenberg model
- Spin Dynamics of bilayer Heisenberg model



A single hole in t-J model

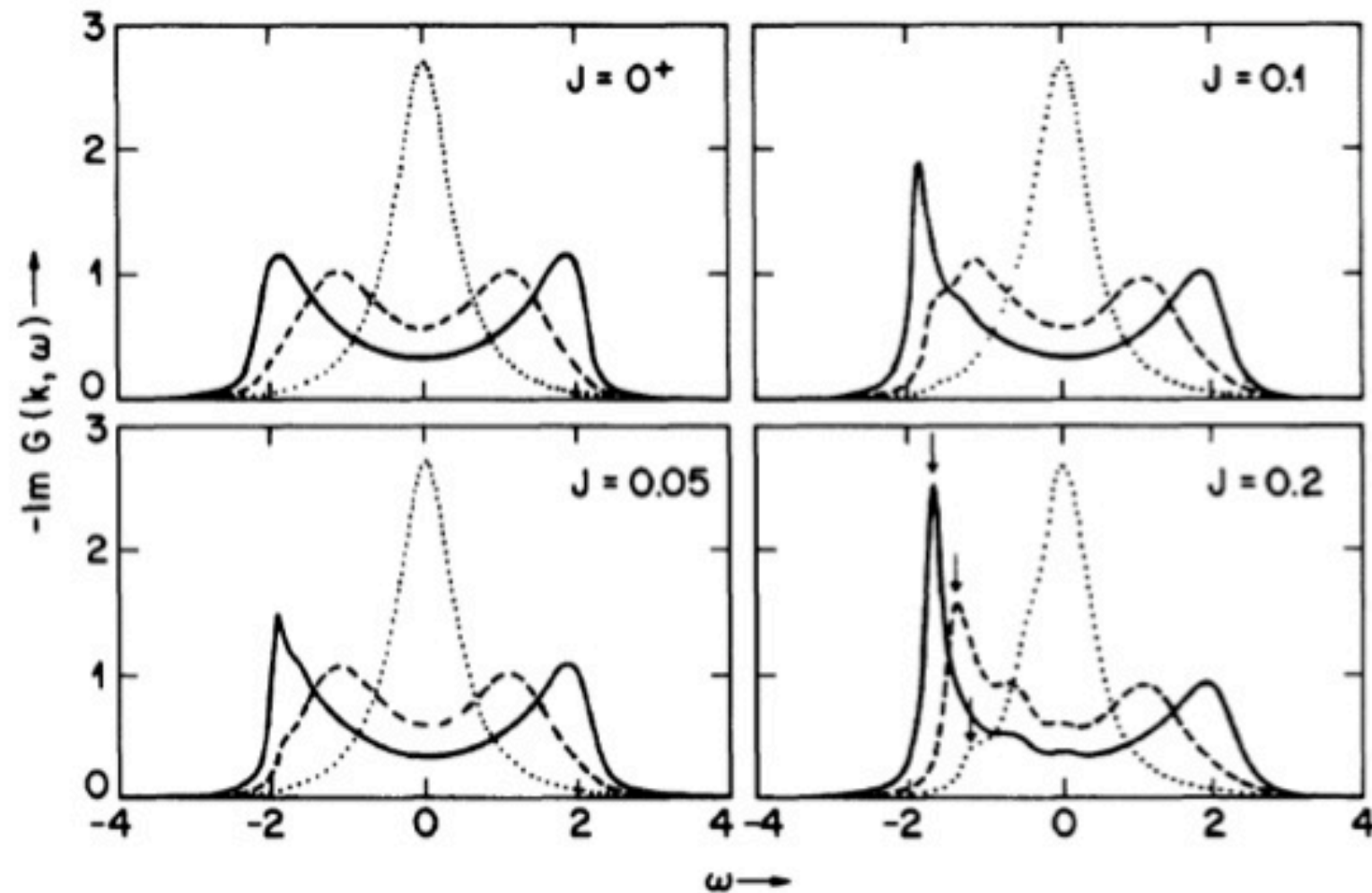
$$H_t = t \sum_{\langle i,j \rangle} h_i^\dagger h_j [b_j^\dagger (1 - b_i^\dagger b_i) + (1 - b_j^\dagger b_j) b_i]$$

$$G(\mathbf{k}, \omega) = \frac{1}{\omega - \sum_{\mathbf{q}} f(\mathbf{k}, \mathbf{q}) G(\mathbf{k} - \mathbf{q}, \omega - E_{\mathbf{q}})}$$



Ref: Schmitt-Rink, et al. PRL 60, 2793 (1988)

A single hole in t-J model



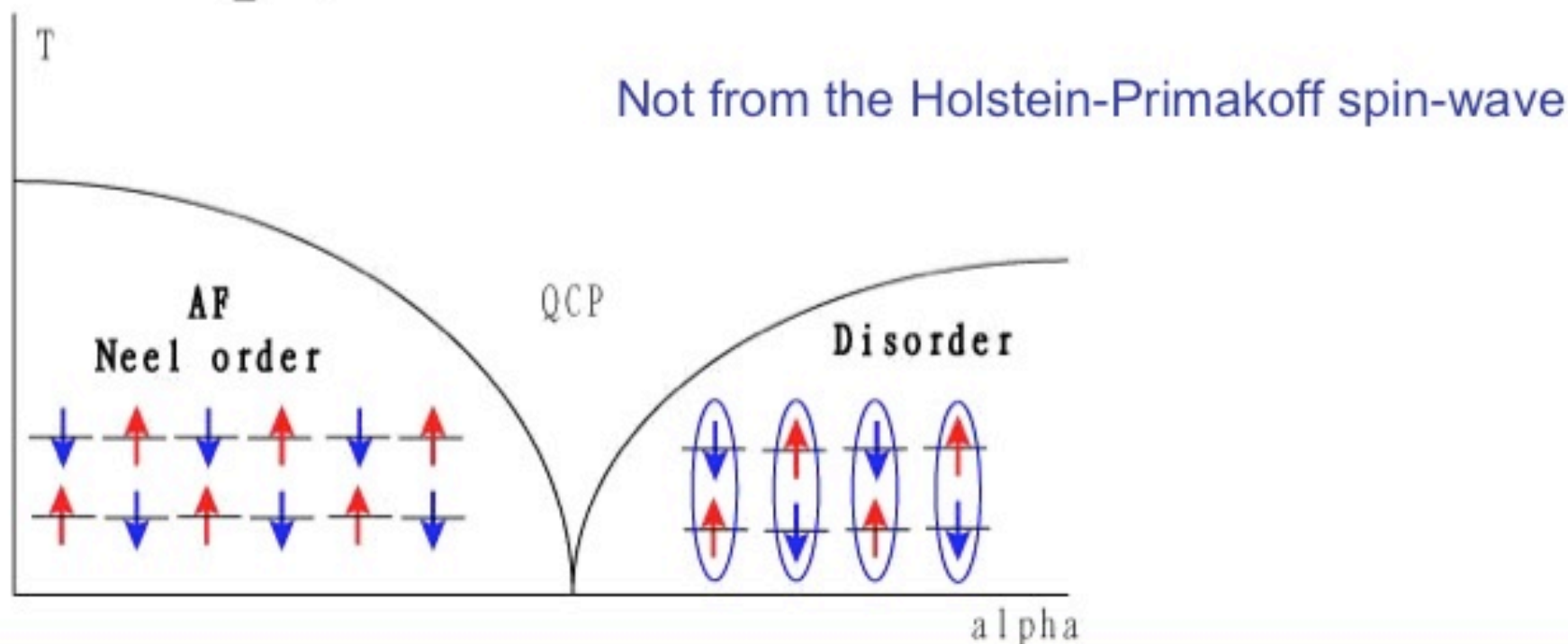
Ref: Schmitt-Rink, et al. PRL 60, 2793 (1988)

Bilayer Heisenberg model

$$H = J_1 \sum_{\langle ij \rangle} (\vec{s}_{i1} \cdot \vec{s}_{j1} + \vec{s}_{i2} \cdot \vec{s}_{j2}) + J_2 \sum_i (\vec{s}_{i1} \cdot \vec{s}_{i2})$$

$$\epsilon_{k,p}^L = J_1 z \sqrt{(\sin^2 2\chi + \alpha \cos 2\chi)(\sin^2 2\chi + \alpha \cos 2\chi \mp \cos^2 2\chi \gamma_k)}$$

$$\epsilon_{k,p}^T = \frac{J_1 z}{2} \sqrt{(\sin^2 2\chi + 2\alpha \cos^2 \chi \mp \cos 2\chi \gamma_k)^2 - \gamma_k^2}$$



Singlet-Triplet Basis and MF Ground State

$$H = J_1 \sum_{\langle ij \rangle} (\vec{s}_{i1} \cdot \vec{s}_{j1} + \vec{s}_{i2} \cdot \vec{s}_{j2}) + J_2 \sum_i (\vec{s}_{i1} \cdot \vec{s}_{i2})$$

$$\vec{S}_i = \vec{s}_{i1} + \vec{s}_{i2} \quad \vec{\tilde{S}} = \vec{s}_{i1} - \vec{s}_{i2}$$

$$A^\dagger = \frac{1}{\sqrt{2}}(c_{1\downarrow}^\dagger c_{2\uparrow}^\dagger - c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger)$$

$$S_z = B_1^\dagger B_1 - B_{-1}^\dagger B_{-1}$$

$$B_1^\dagger = c_{1\uparrow}^\dagger c_{2\uparrow}^\dagger$$

$$S^+ = \sqrt{2}(B_1^\dagger B_0 + B_0 B_{-1}^\dagger)$$

$$B_0^\dagger = \frac{1}{\sqrt{2}}(c_{1\downarrow}^\dagger c_{2\uparrow}^\dagger + c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger)$$

$$\tilde{S}_z = -A^\dagger B_0 - A B_0^\dagger$$

$$\tilde{S}^+ = \sqrt{2}(B_1^\dagger A + A B_{-1}^\dagger)$$

$$B_{-1}^\dagger = c_{1\downarrow}^\dagger c_{2\downarrow}^\dagger$$

$$H = \frac{1}{2} \sum_{\langle ij \rangle} (\vec{S}_i \cdot \vec{S}_j + \vec{\tilde{S}}_i \cdot \vec{\tilde{S}}_j) + \frac{1}{4} \sum_i (\vec{S}_i^2 - \vec{\tilde{S}}_i^2)$$

Singlet-Triplet Basis and MF Ground State

Mean-field order parameter: $\langle \vec{S}_i^\dagger \rangle \equiv \eta_i \tilde{m}$

$$G_i = \eta_i A_i \cos \chi - B_{i0} \sin \chi$$

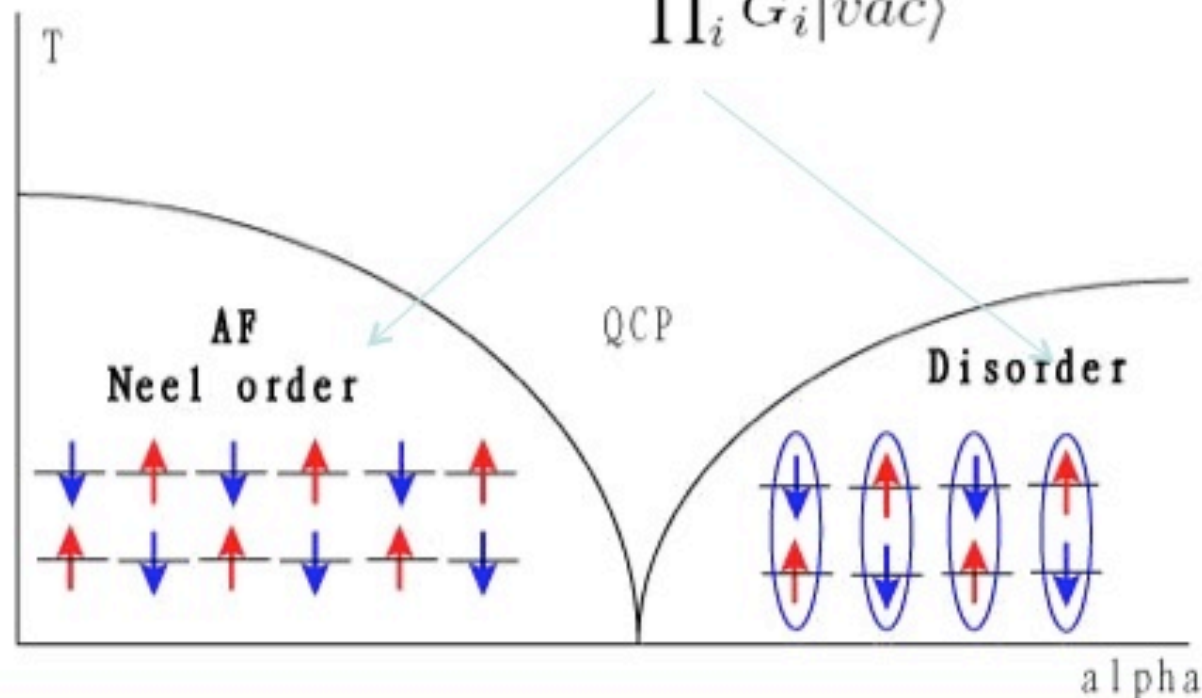
$$\tilde{m} = \sin 2\chi$$

$$E_i = \eta_i A_i \sin \chi + B_{i0} \cos \chi$$

$$\sin 2\chi (\cos 2\chi - \alpha) = 0$$

Mean-field Ground state:

$$\prod_i G_i |vac\rangle$$



Spin-Wave Theory in the Singlet-Triplet Basis

$$\tilde{S}_i^z = \eta_i [\sin 2\chi (1 - 2e_i^\dagger e_i - \sum_{\sigma=\pm 1} b_{i\sigma,p}^\dagger b_{i\sigma}) - \cos 2\chi (e_i^\dagger + e_i)]$$

$$e_i^\dagger = E_i^\dagger G_i \quad \tilde{N}_i = \cos 2\chi (1 - e_i^\dagger e_i - \sum_{\sigma=\pm 1} b_{i\sigma}^\dagger b_{i\sigma}) + \sin(2\chi)(e_i^\dagger + e_i)$$

$$b_{i\pm 1}^\dagger = B_{i\pm 1}^\dagger G_i$$

$$\tilde{S}_i^+ = \sqrt{2}\eta_i [\cos \chi (b_{i+1}^\dagger - b_{i-1}) + \sin \chi (b_{i+1}^\dagger e_i - b_{i-1} e_i^\dagger)]$$

$$S_i^+ = -\sqrt{2} [\sin \chi (b_{i+1}^\dagger + b_{i-1}) - \cos \chi (b_{i+1}^\dagger e_i + b_{i-1} e_i^\dagger)]$$

$$b_{k\sigma,+} = \frac{1}{\sqrt{2}}(b_{k\sigma A}^\dagger + e_{k\sigma B}^\dagger) \quad b_{k\sigma,-} = \frac{1}{\sqrt{2}}(b_{k\sigma A}^\dagger - b_{k\sigma B}^\dagger)$$

$$H^L = J_1 z \sum_{k,p} \{ [-(\sin^2 2\chi + \alpha \cos 2\chi) + p\gamma_k \cos^2 2\chi] e_{k,p}^\dagger e_{k,p} + p\frac{\gamma_k}{2} \cos^2 2\chi (e_{k,p}^\dagger e_{-k,p}^\dagger + h.c.) \}$$

$$H^T = \frac{J_1 z}{2} \sum_{k,\sigma,p} \{ [\sin^2 2\chi - \alpha(1 - \cos 2\chi) - p \cos 2\chi \gamma_k] b_{k\sigma,p}^\dagger b_{k\sigma,p} + p\frac{\gamma_k}{2} (b_{k\sigma,p}^\dagger b_{k-\sigma,p}^\dagger + b_{k\sigma,p} b_{k-\sigma,p}) \}$$

Bogoliubov Transformation

$$\zeta_{k,p}^\dagger = \cosh \varphi_{k,p} e_{k,p}^\dagger + \sinh \varphi_{k,p} e_{-k,p}$$

$$\alpha_{k,p}^\dagger = \cosh \theta_k b_{k1,p}^\dagger + \sinh \theta_{k,p} b_{-k-1,p}$$

$$\beta_{k,p}^\dagger = \cosh \theta_k b_{k-1,p}^\dagger + \sinh \theta_{k,p} b_{-k1,p}$$

$$\tanh 2\varphi_{k,\pm} = -\frac{\mp \frac{1}{2} \gamma_k \cos^2 2\chi}{\sin^2 2\chi + \alpha \cos 2\chi \mp \frac{1}{2} \cos^2 \chi \gamma_k}$$

$$\tanh 2\theta_{k,\pm} = -\frac{\pm \gamma_k}{\sin^2 2\chi + 2\alpha \cos^2 \chi \mp \cos 2\chi \gamma_k}$$

$$\epsilon_{k,p}^L = J_1 z \sqrt{(\sin^2 2\chi + \alpha \cos 2\chi)(\sin^2 2\chi + \alpha \cos 2\chi \mp \cos^2 2\chi \gamma_k)}$$

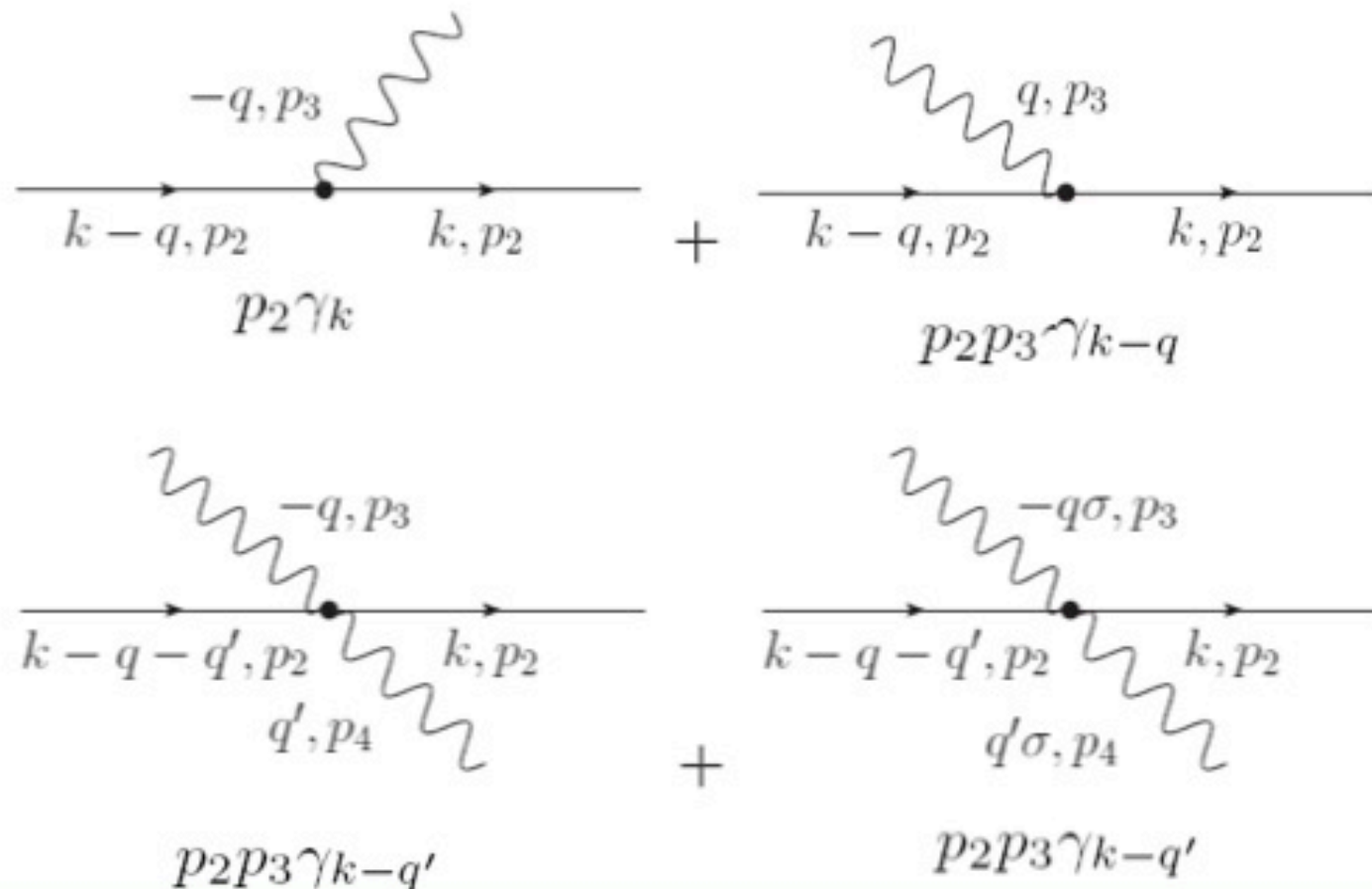
$$\epsilon_{k,p}^T = \frac{J_1 z}{2} \sqrt{(\sin^2 2\chi + 2\alpha \cos^2 \chi \mp \cos 2\chi \gamma_k)^2 - \gamma_k^2}$$

Effective Hamiltonian for a single exciton

$$H = t_e \sum_{i \in A, \delta} E_{i+\delta}^\dagger E_i [\cos 2\chi (1 - e_i^\dagger e_{i+\delta}) + \sin 2\chi (e_i^\dagger + e_{i+\delta}) - \sum_{\sigma} b_{i\sigma}^\dagger b_{i+\delta\sigma}] + h.c.$$

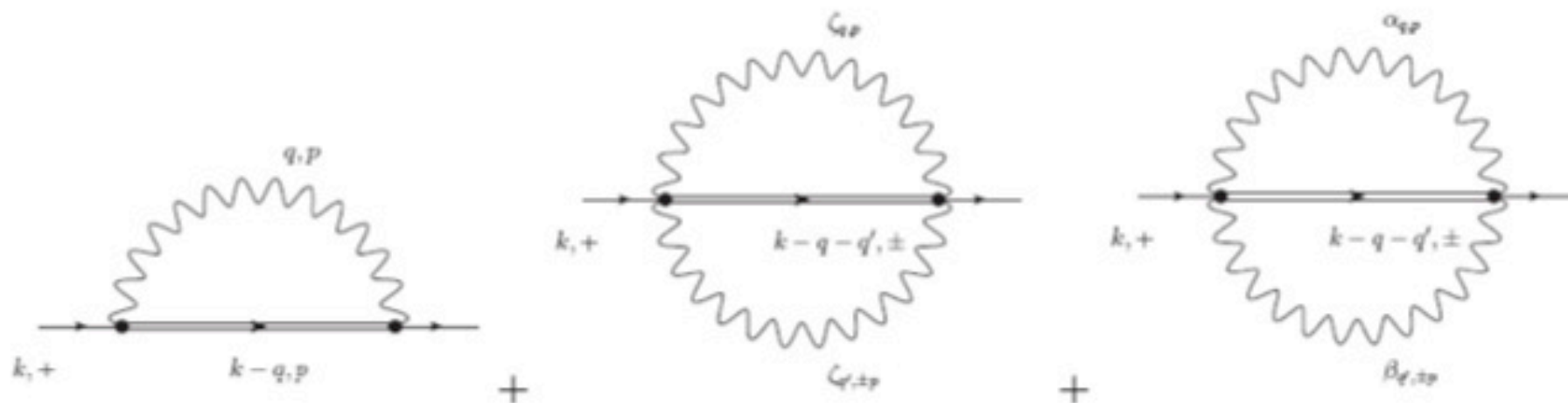
Feynman Diagrams

$$\delta(\Pi_{i=1}^3 p_i - 1)$$



Self-consistent Born Approximation

$$\Sigma^+(k, \omega) =$$

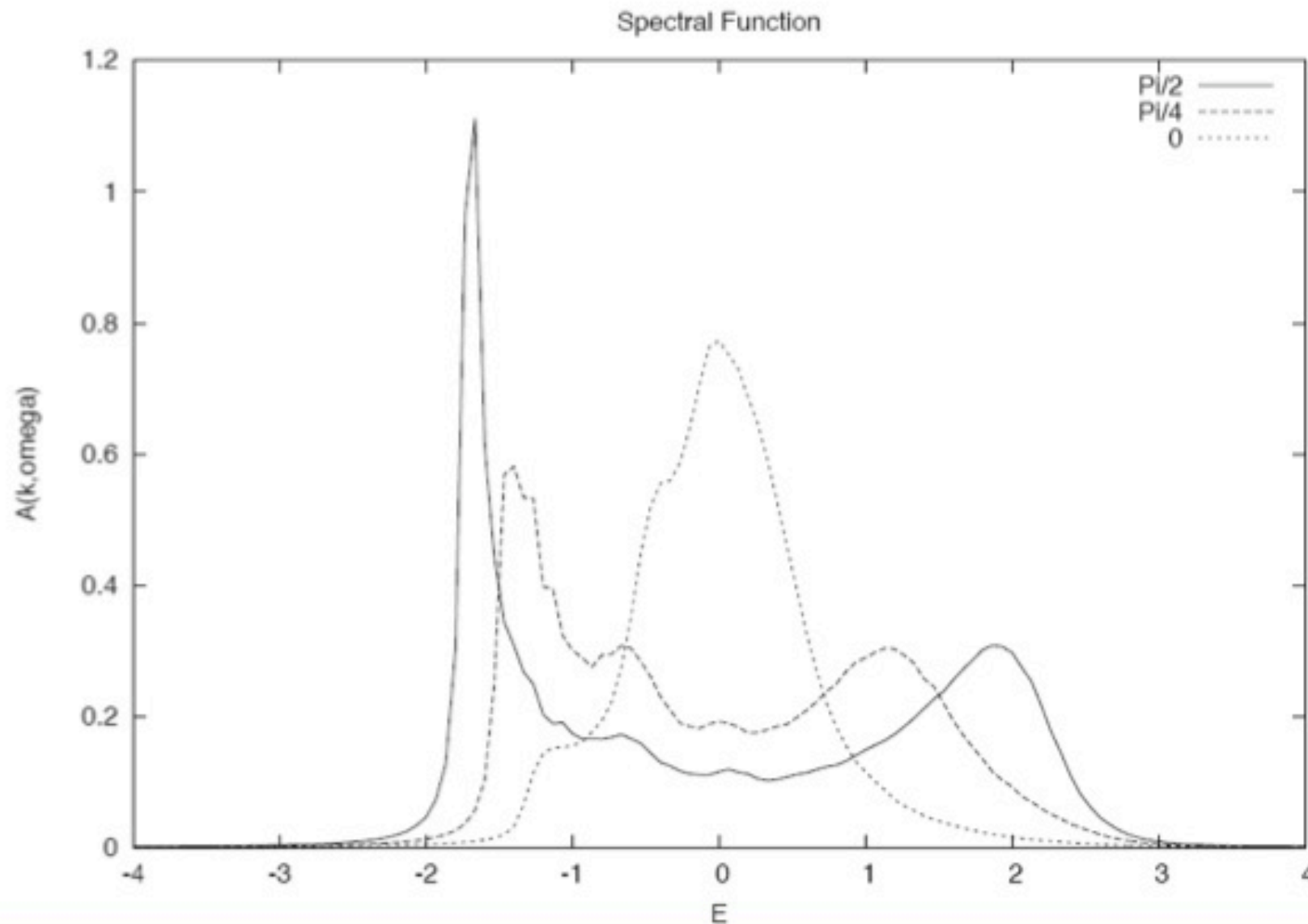


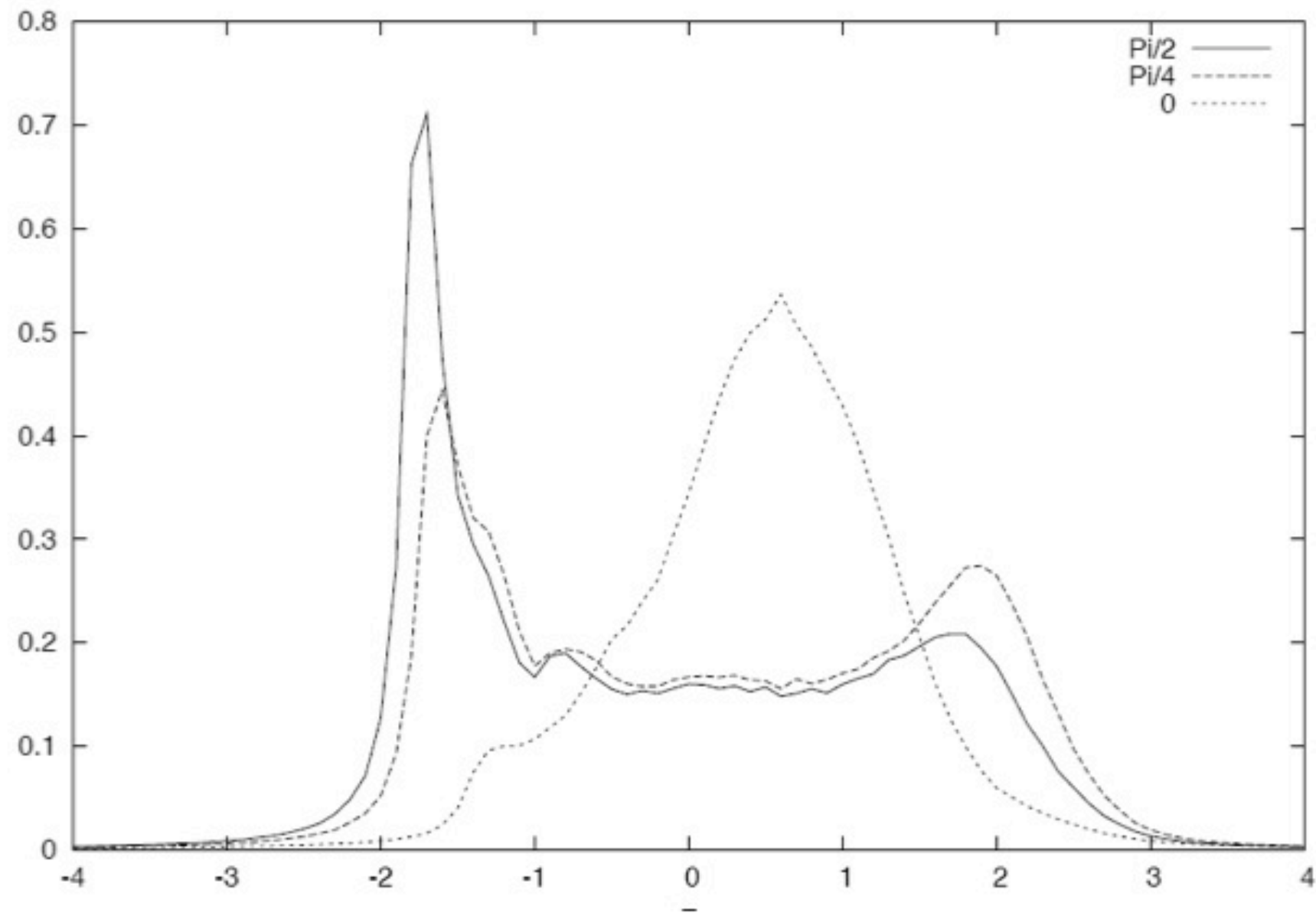
$$(\gamma_{k-q} \cosh \varphi_{q,p} + p \gamma_k \sinh \varphi_{q,p})^2$$

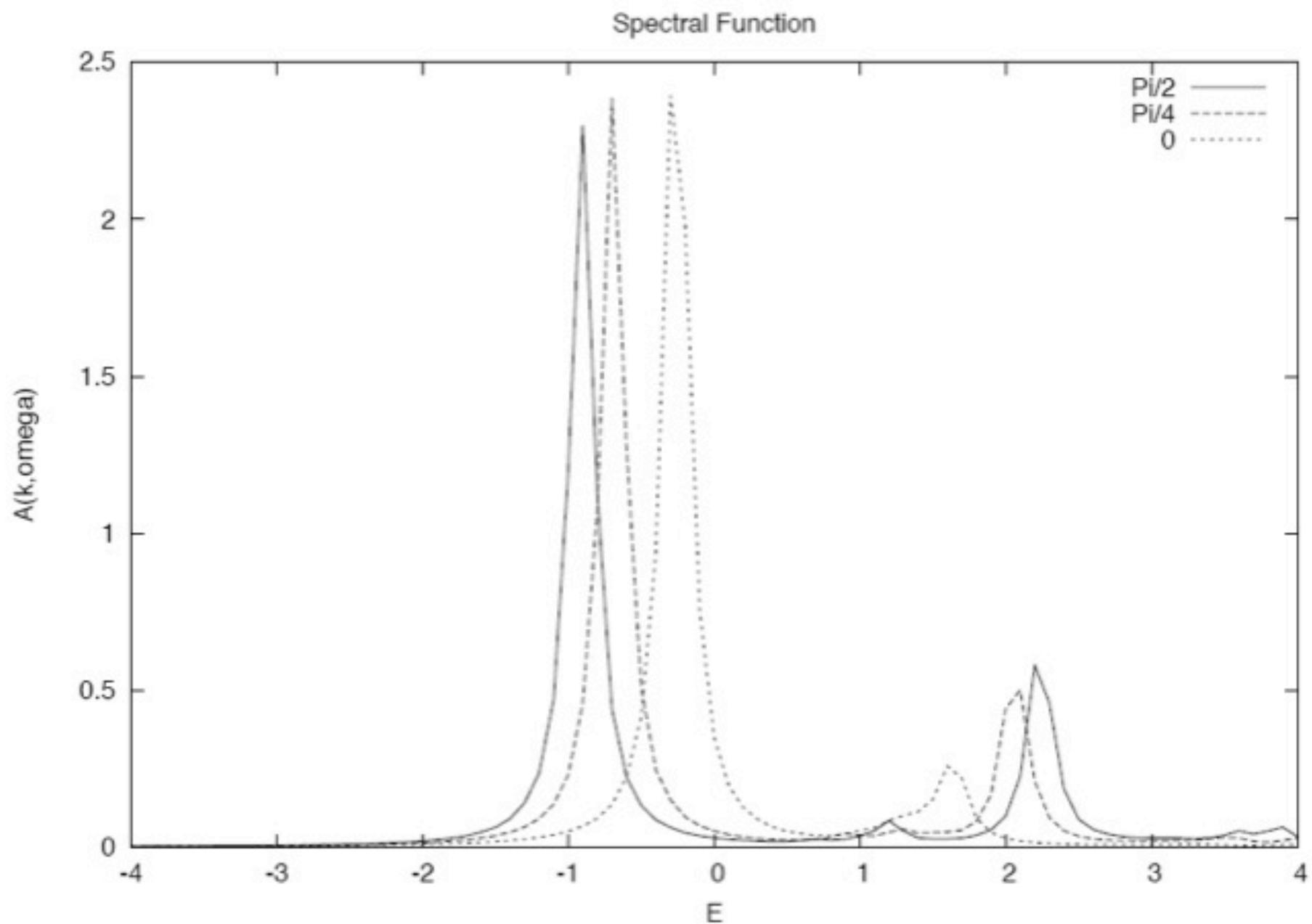
$$(\gamma_{k+q'} \cosh \varphi_{q,p_3} \sinh \varphi_{q',p_4} \pm \gamma_{k+q} \cosh \varphi_{q',p_4} \sinh \varphi_{q,p_3})^2$$

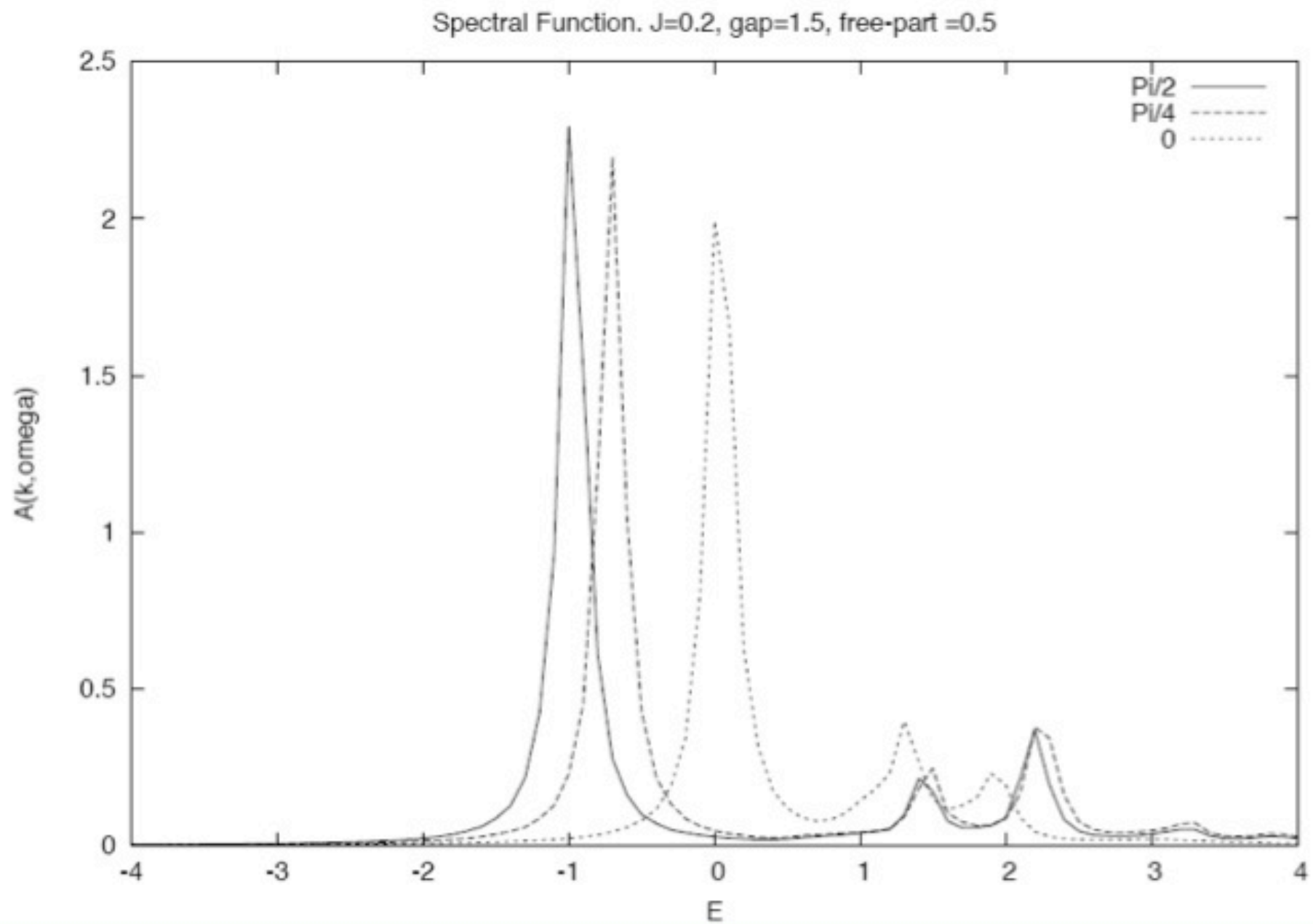
$$(\gamma_{k-q} \cosh \theta_{q,p} \sinh \theta_{q',\pm p} \pm \gamma_{k-q'} \cosh \theta_{q',\pm p} \sinh \theta_{q,p})^2$$

Simulation in the single layer case

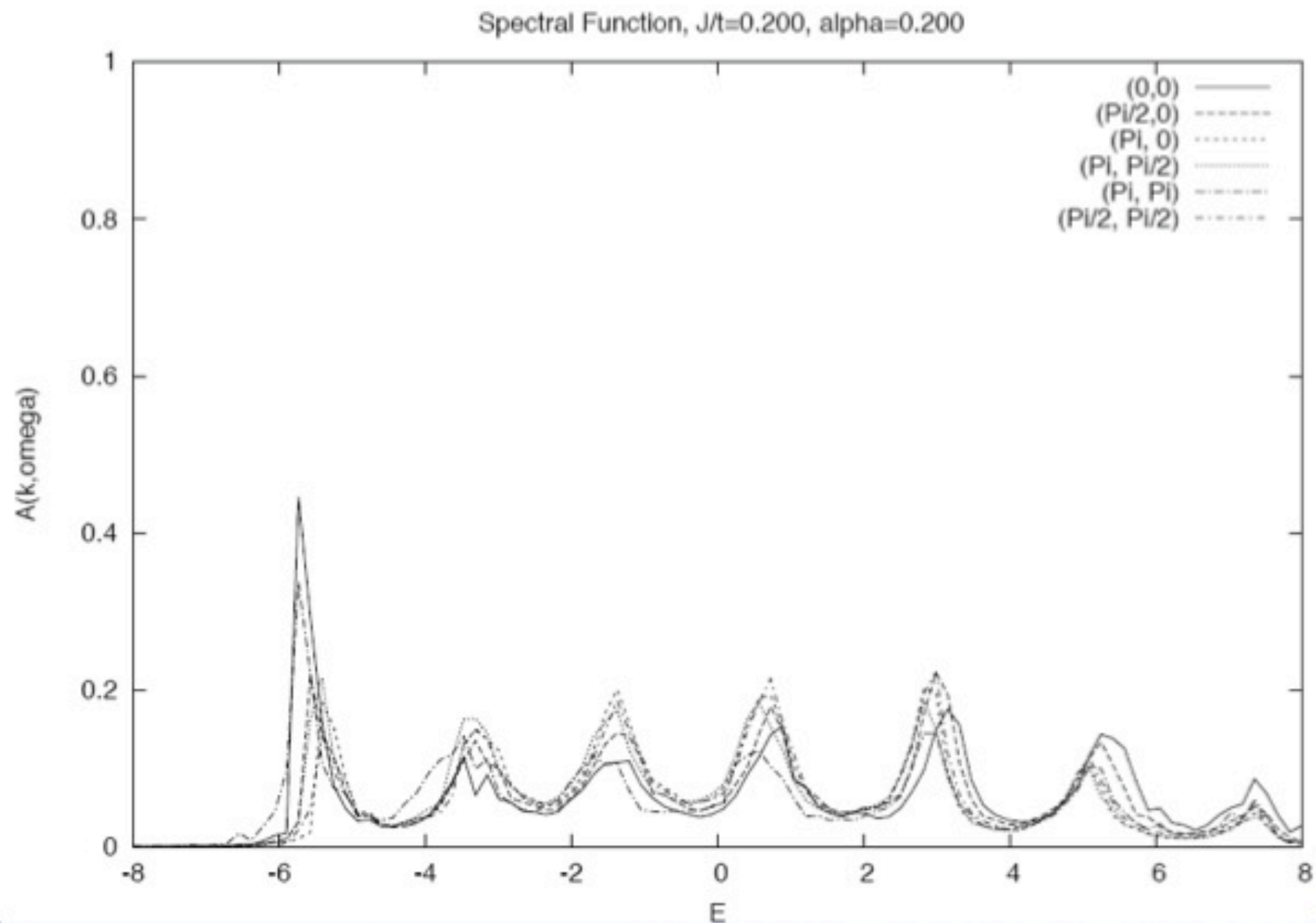


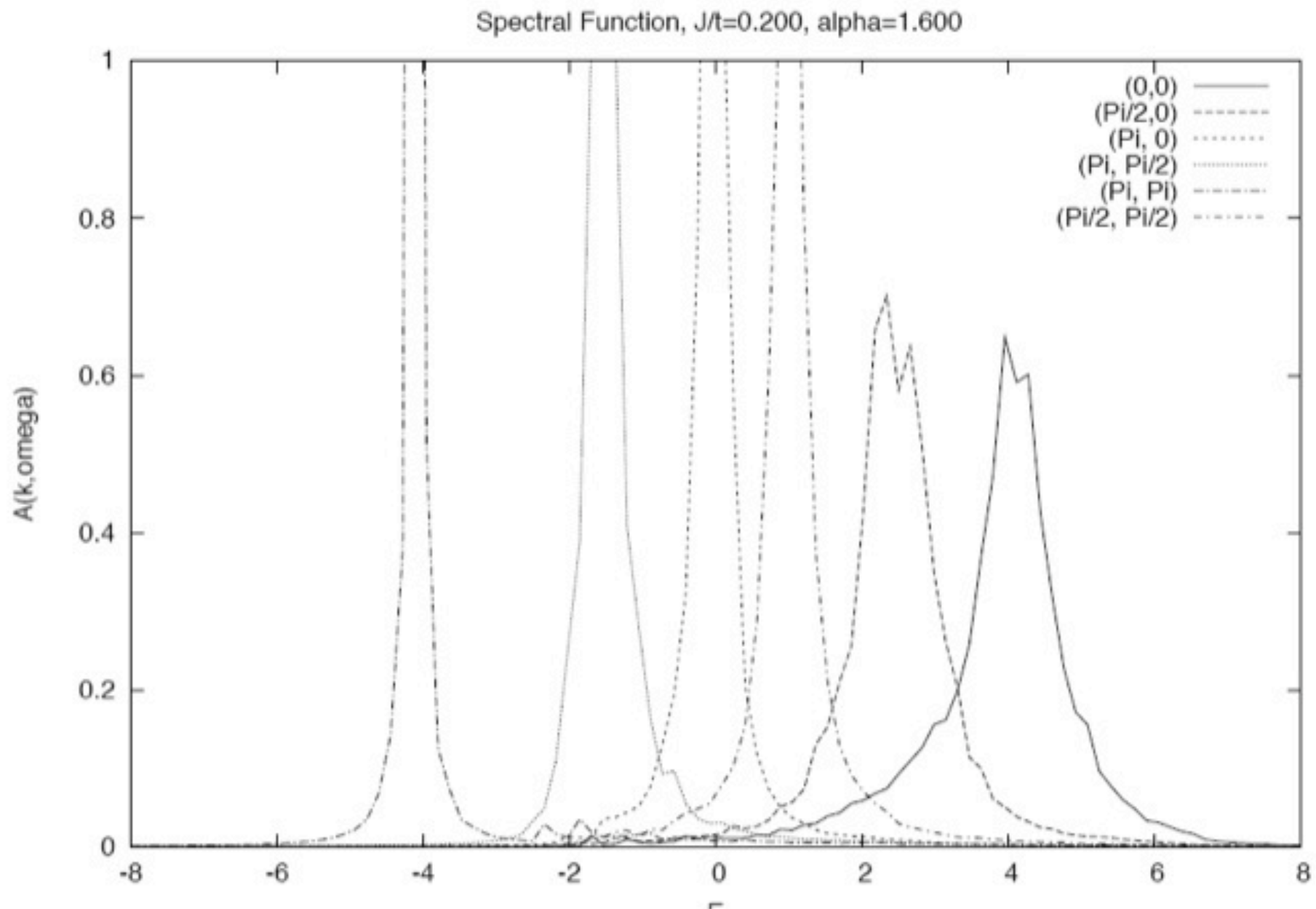




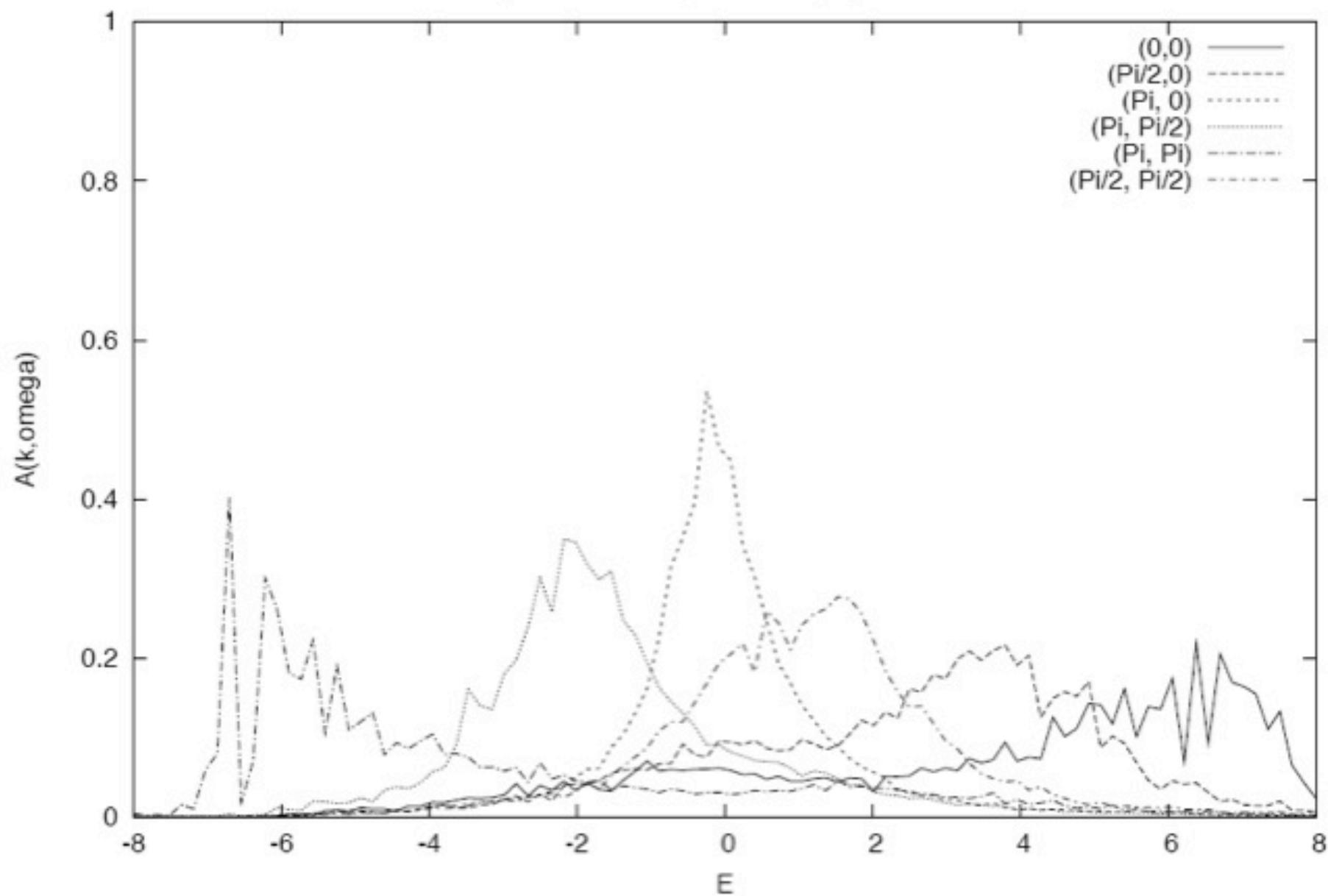


Bilayer numerical results





Spectral Function, $J/t=0.200$, $\alpha=1.000$



Thanks to ...



Prof. Hans Hilgenkamp
(University of Twente)



Prof. Jan Zaanen
(Lorentz Institute, Leiden)

... and you for your attention!



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