The motion of a single exciton in a bilayer quantum antiferromagnet

Louk Rademaker, Kai Wu

Universiteit Leiden
The Netherlands

Overview

- Review: excitons and heterostructures

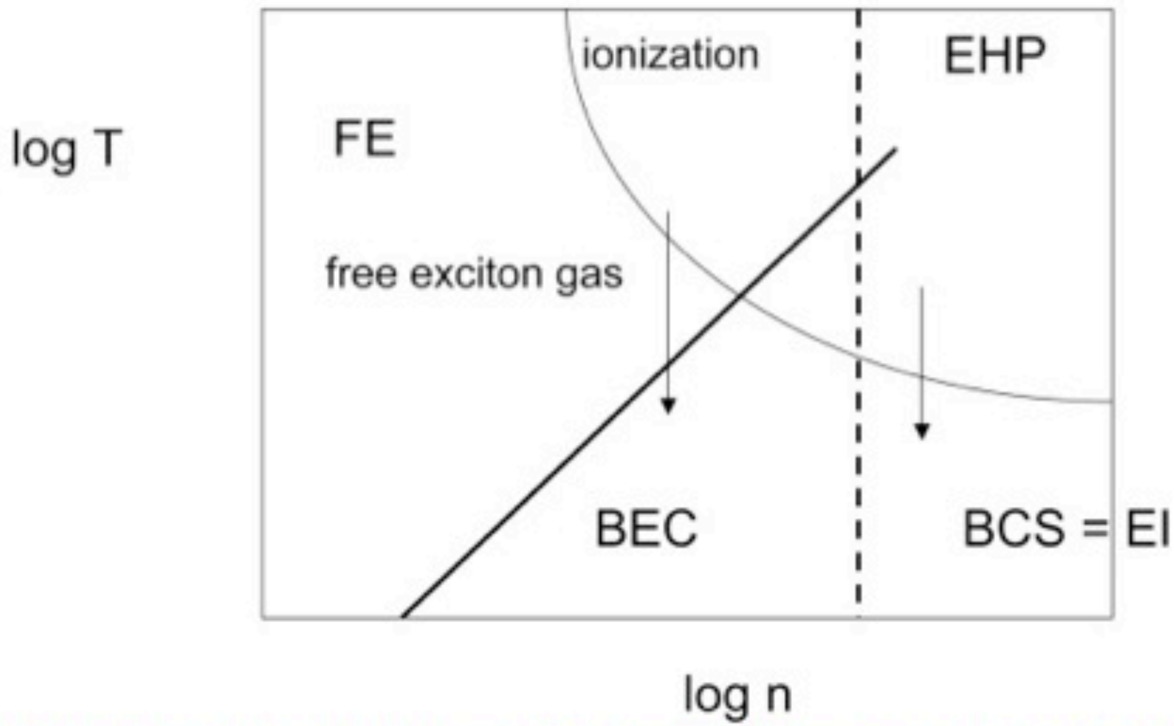
- Holes in strongly correlated layers

- Strongly correlated bilayers

- Excitons in strongly correlated bilayers

Excitons: a little theory

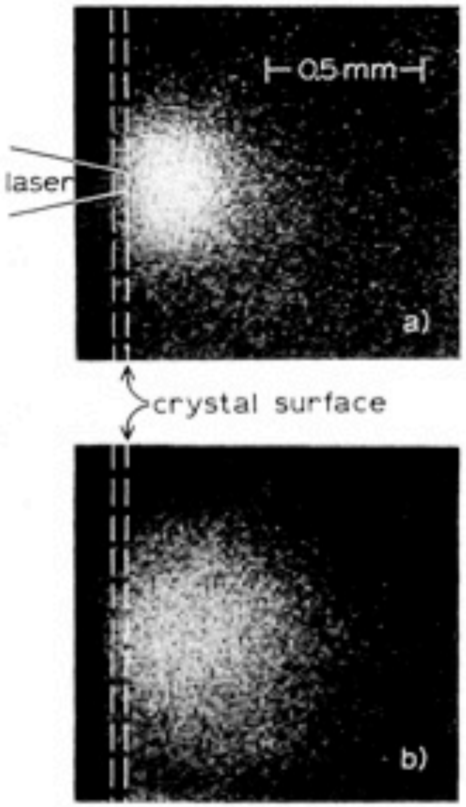
- Bound electron-hole pair
- 'Hydrogen-like' with Bohr radius: $a = \frac{\hbar^2 \varepsilon}{e^2 \mu}$
- General phase diagram:



Ref: Snoke, cond-mat/1011.3844; Moskalenko & Snoke, "BEC of Excitions" (2000)

Excitons: experiments

- Bulk laser excited excitons in Cu_2O

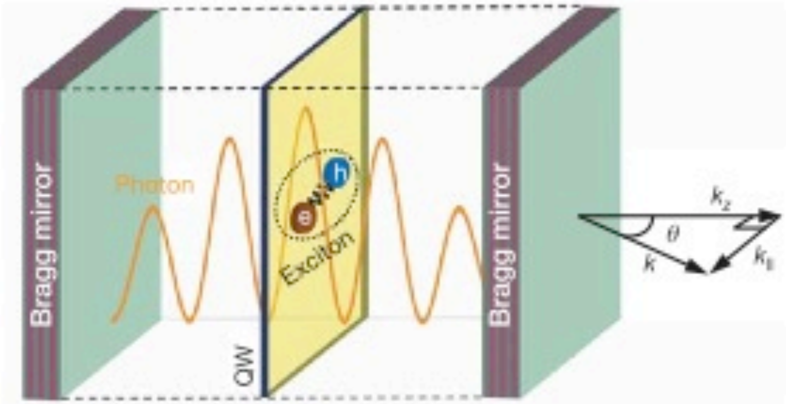


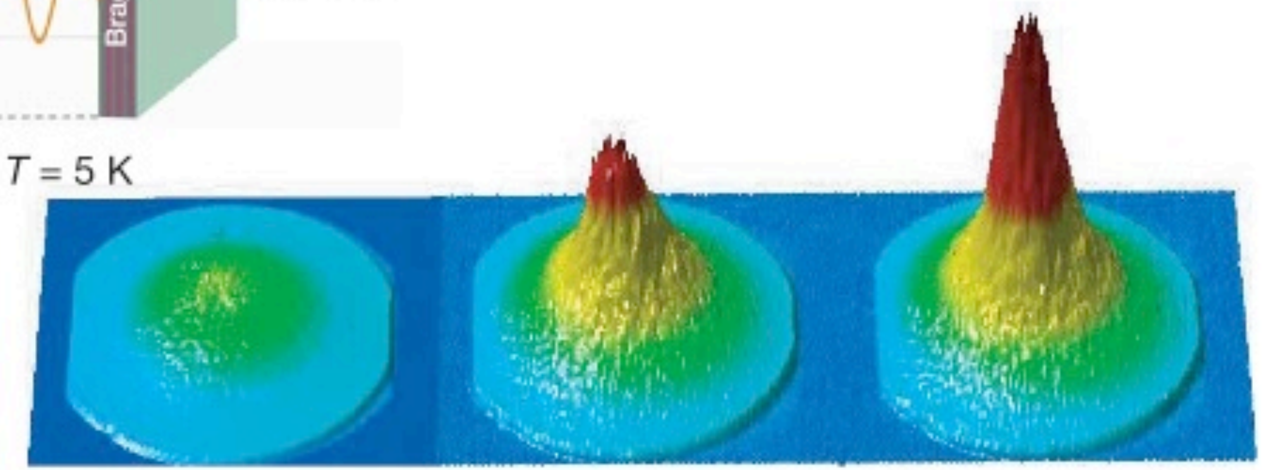
Spectral function > BEC?
Transport > Superfluidity?

Ref: Trauernicht et al, PRL 52, 855 (1984)

Excitons: experiments

- Trapped exciton-polaritons
- BEC at 19 K, coherence proven

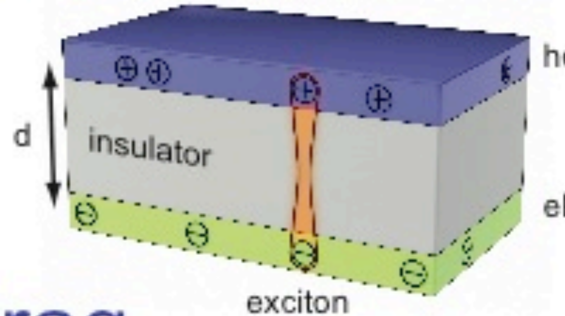




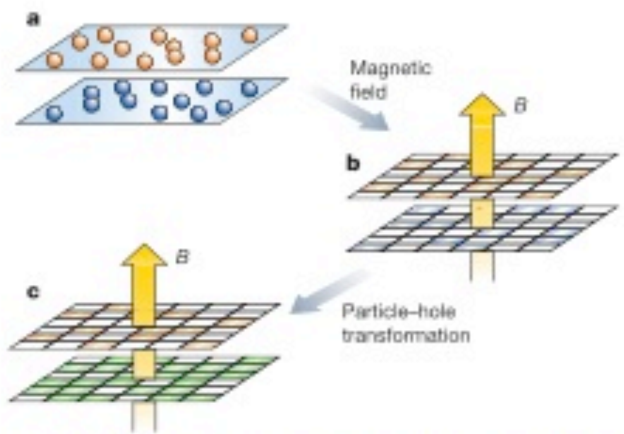
Ref: Kasprzak et al, Nature 443, 409 (2006)

Double layer excitons

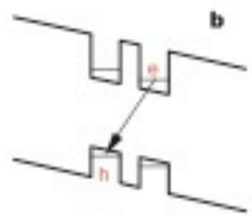
- Double layer excitons
- Features:
  - Tunneling, zero Hall, drag
- Experimental realization:



Quantum Hall Bilayers



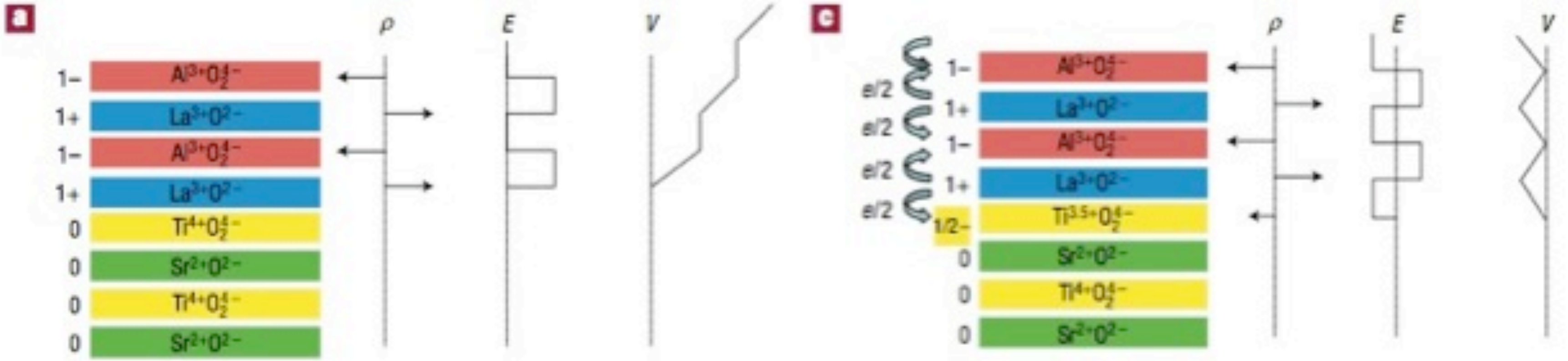
Coupled Quantum Wells



Ref: Eisenstein & MacDonald, Nature 432, 691 (2004);
Butov et al, Nature 418, 751 (2002)

Detour: Oxide Heterostructures

- LAO/STO interface conductivity
- Electronic reconstruction



- Twente 2011: NCCO/LSCO interfaces

Ref: Ohtomo & Hwang, Nature 427, 423 (2004);
Nakagawa et al, Nat. Mater. 5, 204 (2006)

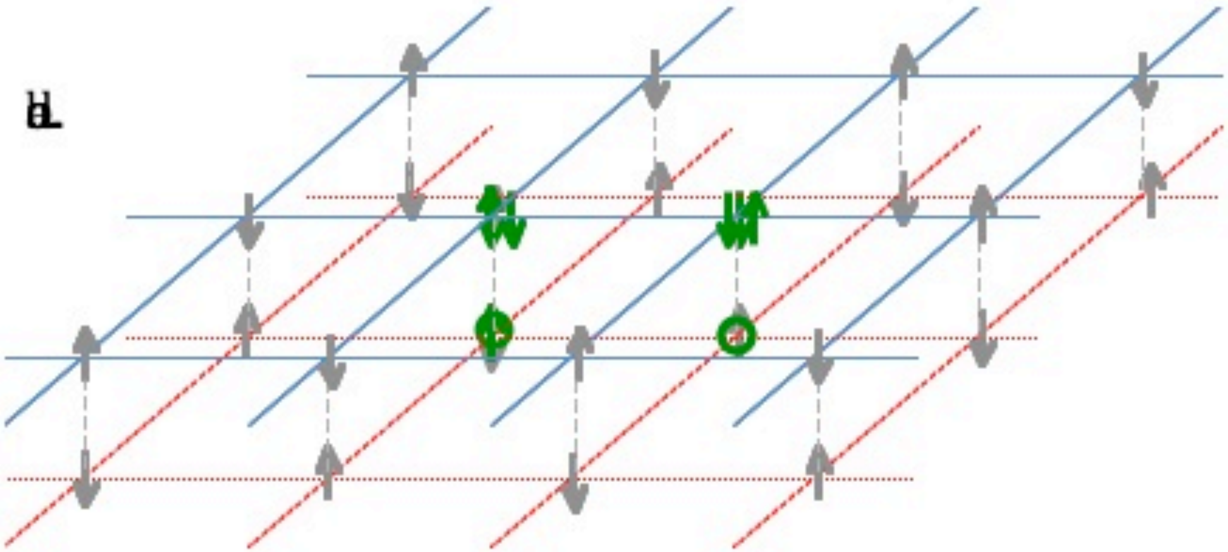
Our research

- What are the properties of excitons in electron-hole strongly correlated oxide heterostructures?

- Problem:
  No (good) theory for the doped Mott insulator

The exciton t-J model

- Assumptions:
  - only interface layers are relevant
  - excitons are bound states



Effective t-J model of Excitons

$$H = H_t + H_J$$

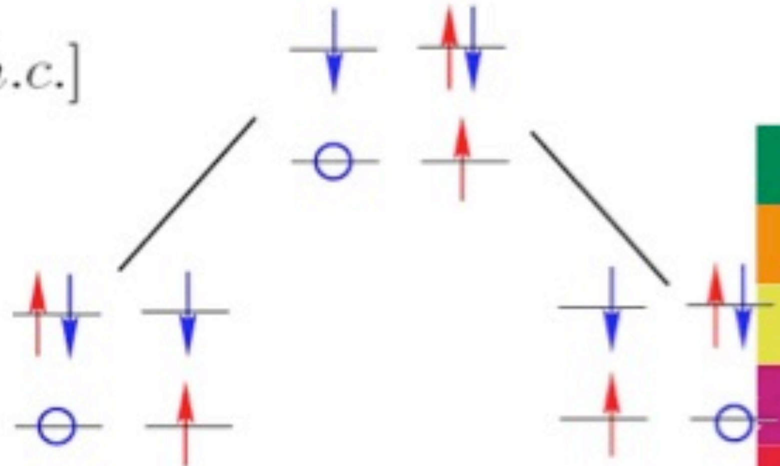
$$H_t = -t \sum_{\langle ij \rangle, \sigma} (c_{i1\sigma}^{\dagger} c_{j1\sigma} + c_{i2\sigma}^{\dagger} c_{j2\sigma} + h.c.) - V \sum_i n_{i1}^d n_{i2}^h$$

$$H_J = J_1 \sum_{\langle ij \rangle} (\vec{\mathbf{s}}_{i1} \cdot \vec{\mathbf{s}}_{j1} + \vec{\mathbf{s}}_{i2} \cdot \vec{\mathbf{s}}_{j2}) + J_2 \sum_i (\vec{\mathbf{s}}_{i1} \cdot \vec{\mathbf{s}}_{i2})$$

Second-order perturbation

$$H_t = -\frac{2t^2}{V} \sum_{\langle ij \rangle \sigma\sigma'} (c_{i1\sigma}^{\dagger} c_{j1\sigma} c_{i2\sigma'} c_{j2\sigma'}^{\dagger} + h.c.)$$

$$= -t_e \sum_{\langle ij \rangle \sigma\sigma'} [(c_{i1\sigma}^{\dagger} c_{i2\sigma'})(c_{j2\sigma'}^{\dagger} c_{j1\sigma}) + h.c.]$$



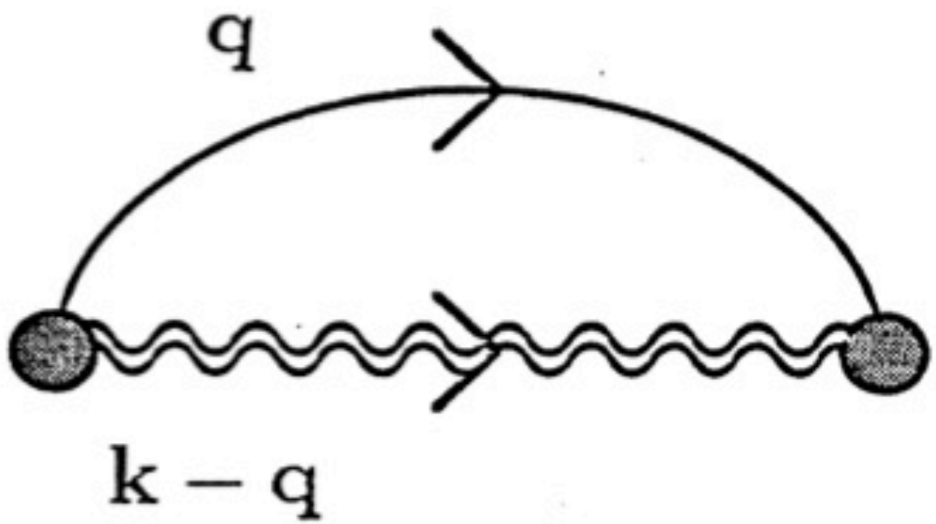
How does a single exciton move in the bilayer antiferromagnet?

- A single hole moves in a quantum antiferromagnet
  - Effective Hamiltonian
  - Self-consistent Born Approximation;
- Effective Hamiltonian for the Exciton from the bilayer Heisenberg model
- Spin Dynamics of bilayer Heisenberg model

A single hole in t-J model

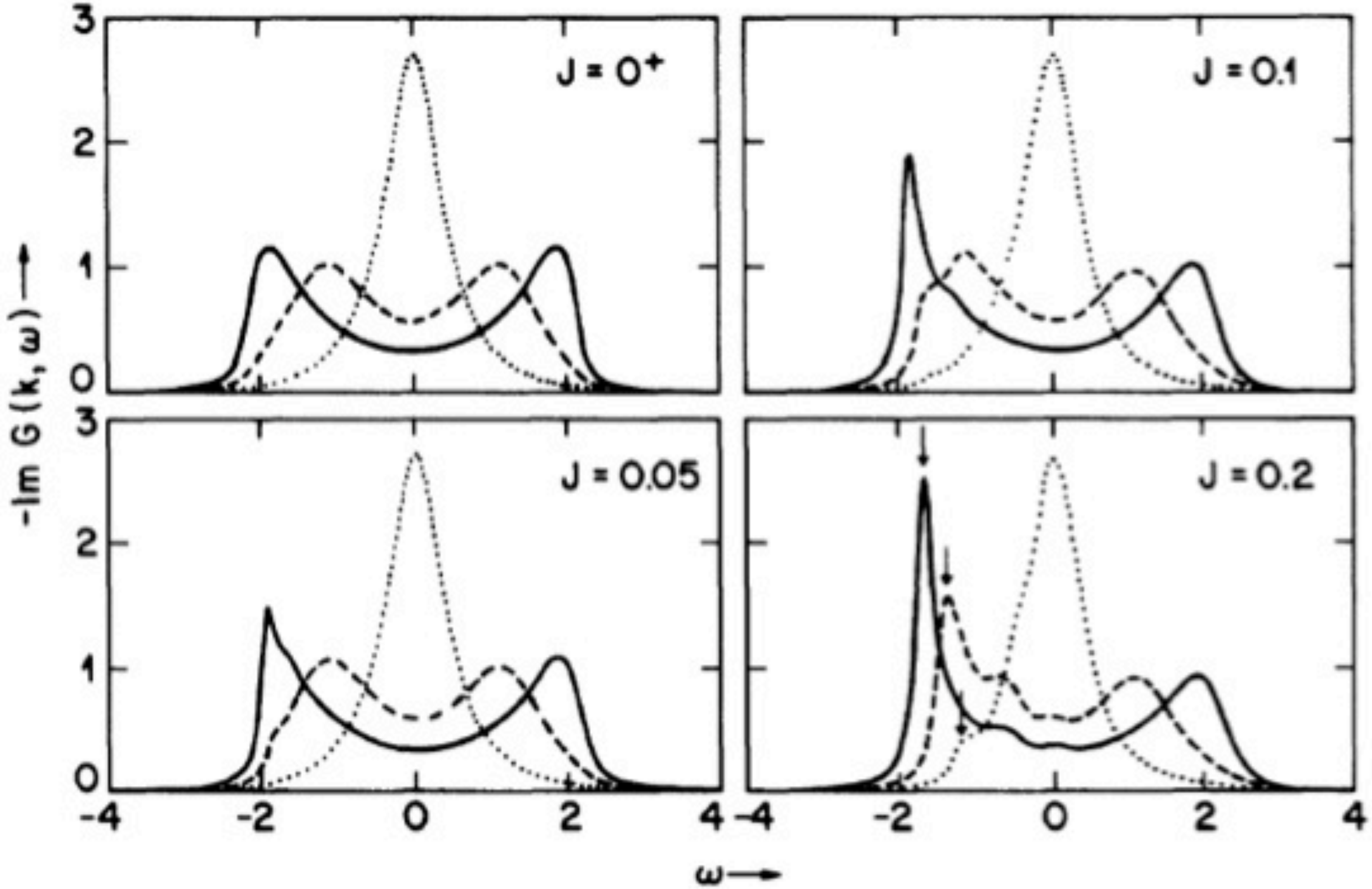
$$H_t = t \sum_{\langle i,j \rangle} h_i^\dagger h_j [b_j^\dagger (1 - b_i^\dagger b_i) + (1 - b_j^\dagger b_j) b_i]$$

$$G(\mathbf{k}, w) = \frac{1}{\omega - \sum_q f(\mathbf{k}, \mathbf{q}) G(\mathbf{k} - \mathbf{q}, \omega - E_{\mathbf{q}})}$$



Ref: Schmitt-Rink, et al. PRL 60, 2793 (1988)

A single hole in t-J model



Ref: Schmitt-Rink, et al. PRL 60, 2793 (1988)

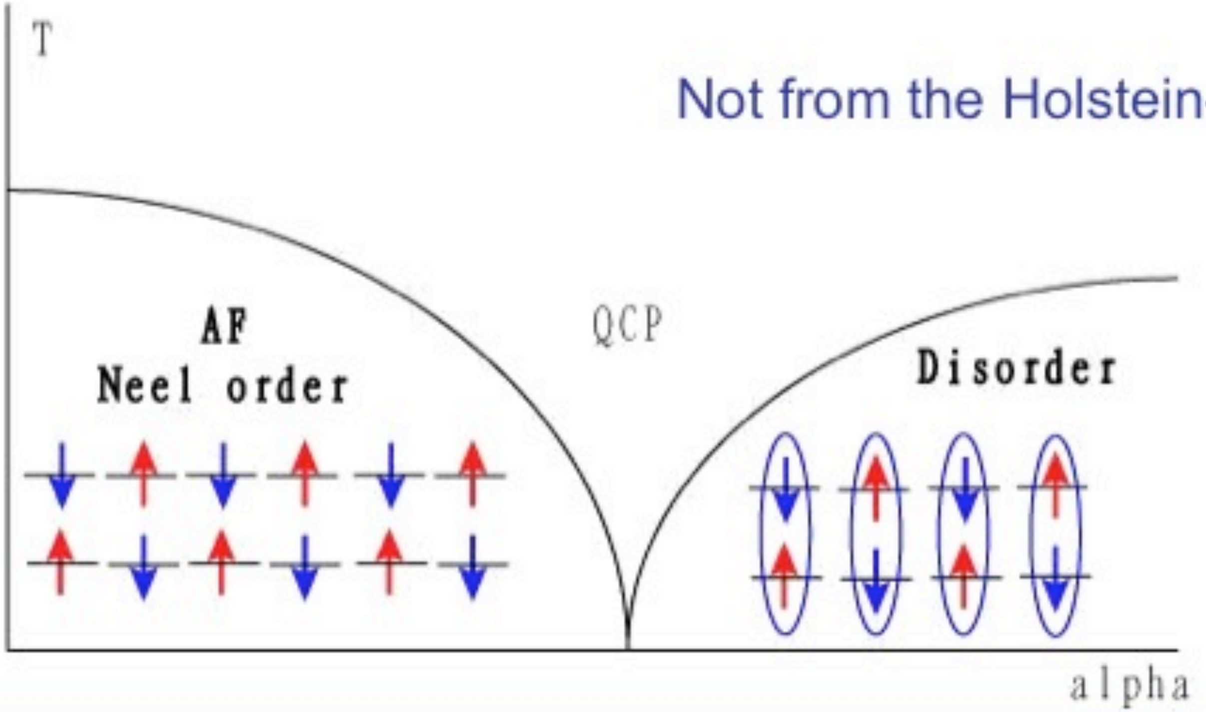
Bilayer Heisenberg model

$$H = J_1 \sum_{\langle ij \rangle} (\vec{s}_{i1} \cdot \vec{s}_{j1} + \vec{s}_{i2} \cdot \vec{s}_{j2}) + J_2 \sum_i (\vec{s}_{i1} \cdot \vec{s}_{i2})$$

$$\epsilon^L_{k,p} = J_1 z \sqrt{(\sin^2 2\chi + \alpha \cos 2\chi)(\sin^2 2\chi + \alpha \cos 2\chi \mp \cos^2 2\chi \gamma_k)}$$

$$\epsilon^T_{k,p} = \frac{J_1 z}{2} \sqrt{(\sin^2 2\chi + 2\alpha \cos^2 \chi \mp \cos 2\chi \gamma_k)^2 - \gamma_k^2}$$

Not from the Holstein-Primakoff spin-wave



Singlet-Triplet Basis and MF Ground State

$$H = J_1 \sum_{\langle ij \rangle} (\vec{s}_{i1} \cdot \vec{s}_{j1} + \vec{s}_{i2} \cdot \vec{s}_{j2}) + J_2 \sum_i (\vec{s}_{i1} \cdot \vec{s}_{i2})$$

$$\vec{S}_i = \vec{s}_{i1} + \vec{s}_{i2} \quad \vec{\tilde{S}} = \vec{s}_{i1} - \vec{s}_{i2}$$

$$A^\dagger = \frac{1}{\sqrt{2}}(c^\dagger_{1\downarrow} c^\dagger_{2\uparrow} - c^\dagger_{1\uparrow} c^\dagger_{2\downarrow})$$

$$B^\dagger_1 = c^\dagger_{1\uparrow} c^\dagger_{2\uparrow}$$

$$B^\dagger_0 = \frac{1}{\sqrt{2}}(c^\dagger_{1\downarrow} c^\dagger_{2\uparrow} + c^\dagger_{1\uparrow} c^\dagger_{2\downarrow})$$

$$B^\dagger_{-1} = c^\dagger_{1\downarrow} c^\dagger_{2\downarrow}$$

$$S_z = B^\dagger_1 B_1 - B^\dagger_{-1} B_{-1}$$

$$S^+ = \sqrt{2}(B^\dagger_1 B_0 + B_0 B^\dagger_{-1})$$

$$\tilde{S}_z = -A^\dagger B_0 - A B^\dagger_0$$

$$\tilde{S}^+ = \sqrt{2}(B^\dagger_1 A + A B^\dagger_{-1})$$

$$H = \frac{1}{2} \sum_{\langle ij \rangle} (\vec{S}_i \cdot \vec{S}_j + \vec{\tilde{S}}_i \cdot \vec{\tilde{S}}_j) + \frac{1}{4} \sum_i (\vec{S}_i^2 - \vec{\tilde{S}}_i^2)$$

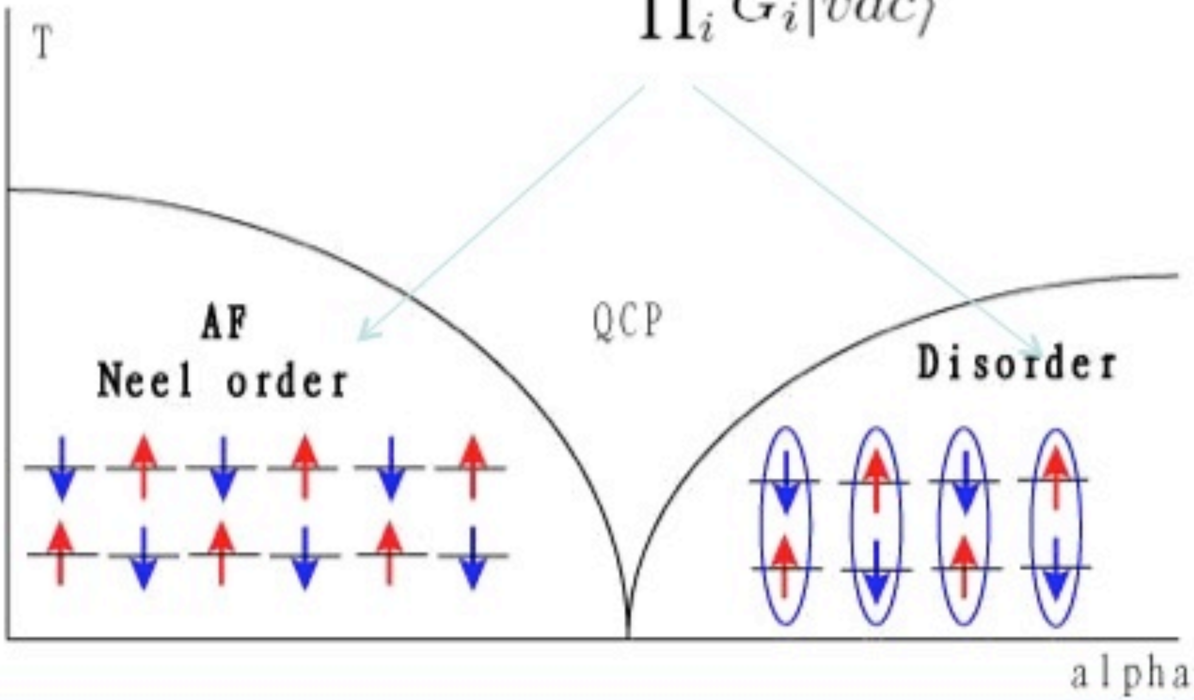
Singlet-Triplet Basis and MF Ground State

Mean-field order parameter: $\langle \vec{\tilde{S}}_i^\dagger \rangle \equiv \eta_i \tilde{m}$

$$G_i = \eta_i A_i \cos\chi - B_{i0} \sin\chi \qquad \tilde{m} = \sin 2\chi$$

$$E_i = \eta_i A_i \sin\chi + B_{i0} \cos\chi \qquad \sin 2\chi(\cos 2\chi - \alpha) = 0$$

Mean-field Ground state: $\prod_i G_i |vac\rangle$



Spin-Wave Theory in the Singlet-Triplet Basis

$$e_i^\dagger = E_i^\dagger G_i$$
$$b_{i\pm 1}^\dagger = B_{i\pm 1}^\dagger G_i$$

$$\tilde{S}_i^z = \eta_i[\sin 2\chi(1 - 2e_i^\dagger e_i - \sum_{\sigma=\pm 1} b_{i\sigma,p}^\dagger b_{i\sigma}) - \cos 2\chi(e_i^\dagger + e_i)]$$
$$\tilde{N}_i = \cos 2\chi(1 - e_i^\dagger e_i - \sum_{\sigma=\pm 1} b_{i\sigma}^\dagger b_{i\sigma}) + \sin(2\chi)(e_i^\dagger + e_i)$$
$$\tilde{S}_i^+ = \sqrt{2}\eta_i[\cos\chi(b_{i+1}^\dagger - b_{i-1}) + \sin\chi(b_{i+1}^\dagger e_i - b_{i-1}e_i^\dagger)]$$
$$S_i^+ = -\sqrt{2}[\sin\chi(b_{i+1}^\dagger + b_{i-1}) - \cos\chi(b_{i+1}^\dagger e_i + b_{i-1}e_i^\dagger)]$$

$$b_{k\sigma,+} = \frac{1}{\sqrt{2}}(b_{k\sigma A}^\dagger + e_{k\sigma B}^\dagger) \quad b_{k\sigma,-} = \frac{1}{\sqrt{2}}(b_{k\sigma A}^\dagger - b_{k\sigma B}^\dagger)$$

$$H^L = J_1 z \sum_{k,p}\{[-(\sin^2 2\chi + \alpha\cos 2\chi) + p\gamma_k\cos^2 2\chi]e_{k,p}^\dagger e_{k,p} + p\frac{\gamma_k}{2}\cos^2 2\chi(e_{k,p}^\dagger e_{-k,p}^\dagger + h.c.)\}$$
$$H^T = \frac{J_1 z}{2}\sum_{k,\sigma,p}\{[\sin^2 2\chi - \alpha(1-\cos 2\chi) - p\cos 2\chi\gamma_k]b_{k\sigma,p}^\dagger b_{k\sigma,p} + p\frac{\gamma_k}{2}(b_{k\sigma,p}^\dagger b_{k-\sigma,p}^\dagger + b_{k\sigma,p}b_{k-\sigma,p})\}$$

Bogoliubov Transformation

$$\zeta^{\dagger}_{k,p} = \cosh\varphi_{k,p} e^{\dagger}_{k,p} + \sinh\varphi_{k,p} e_{-k,p}$$
$$\alpha^{\dagger}_{k,p} = \cosh\theta_k b^{\dagger}_{k1,p} + \sinh\theta_{k,p} b_{-k-1,p}$$
$$\beta^{\dagger}_{k,p} = \cosh\theta_k b^{\dagger}_{k-1,p} + \sinh\theta_{k,p} b_{-k1,p}$$

$$\tanh 2\varphi_{k,\pm} = -\frac{\mp\frac{1}{2}\gamma_k \cos^2 2\chi}{\sin^2 2\chi + \alpha\cos 2\chi \mp \frac{1}{2}\cos^2\chi\gamma_k}$$
$$\tanh 2\theta_{k,\pm} = -\frac{\pm\gamma_k}{\sin^2 2\chi + 2\alpha\cos^2\chi \mp \cos 2\chi\gamma_k}$$

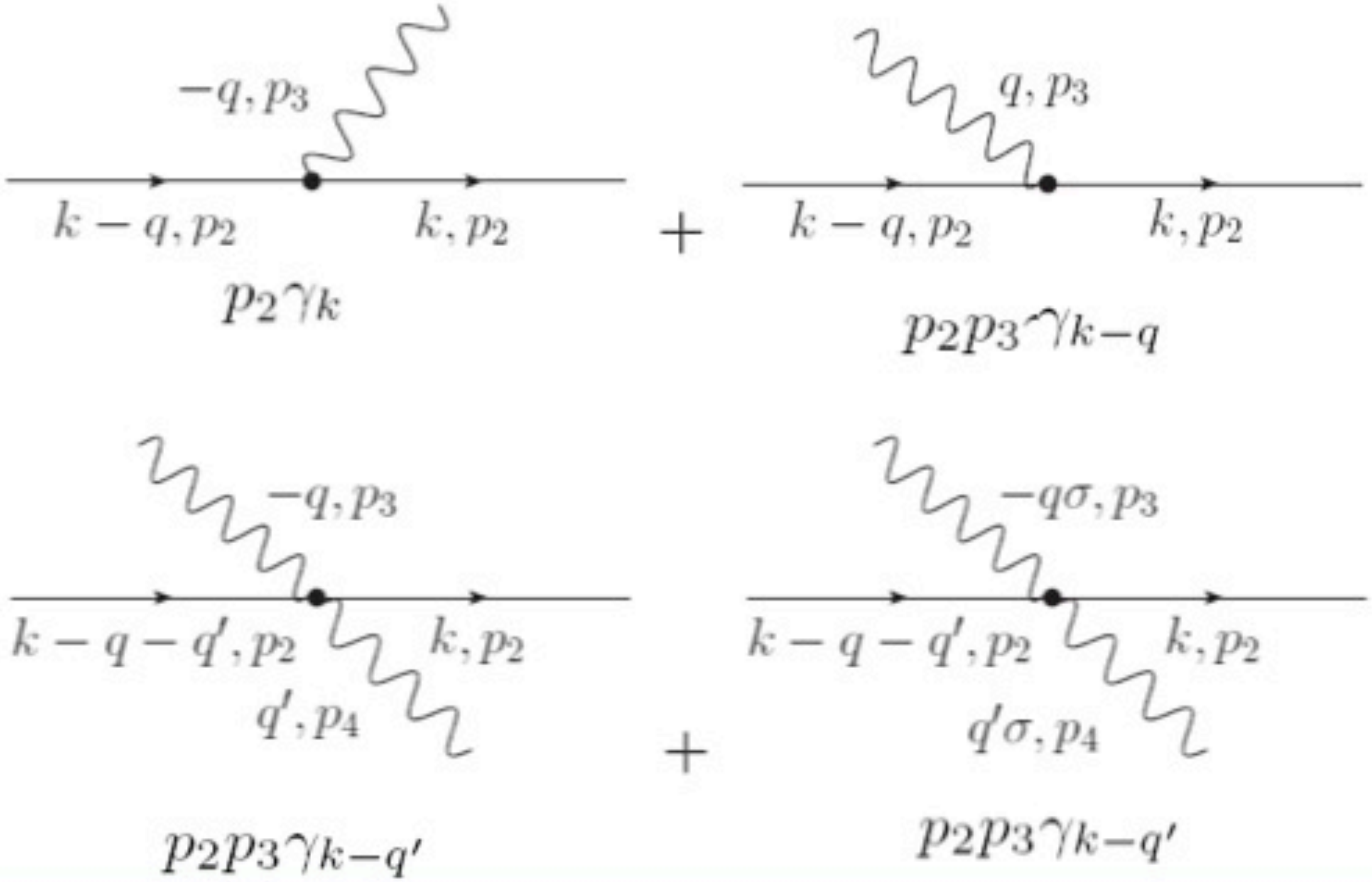
$$\epsilon^{L}_{k,p} = J_1 z\sqrt{(\sin^2 2\chi + \alpha\cos 2\chi)(\sin^2 2\chi + \alpha\cos 2\chi \mp \cos^2 2\chi\gamma_k)}$$
$$\epsilon^{T}_{k,p} = \frac{J_1 z}{2}\sqrt{(\sin^2 2\chi + 2\alpha\cos^2\chi \mp \cos 2\chi\gamma_k)^2 - \gamma_k^2}$$

Effective Hamiltonian for a single exciton

$$H = t_e \sum_{i \in A, \delta} E^{\dagger}_{i+\delta} E_i [\cos 2\chi (1 - e^{\dagger}_i e_{i+\delta}) + \sin 2\chi (e^{\dagger}_i + e_{i+\delta}) - \sum_{\sigma} b^{\dagger}_{i\sigma} b_{i+\delta\sigma}] + h.c.$$

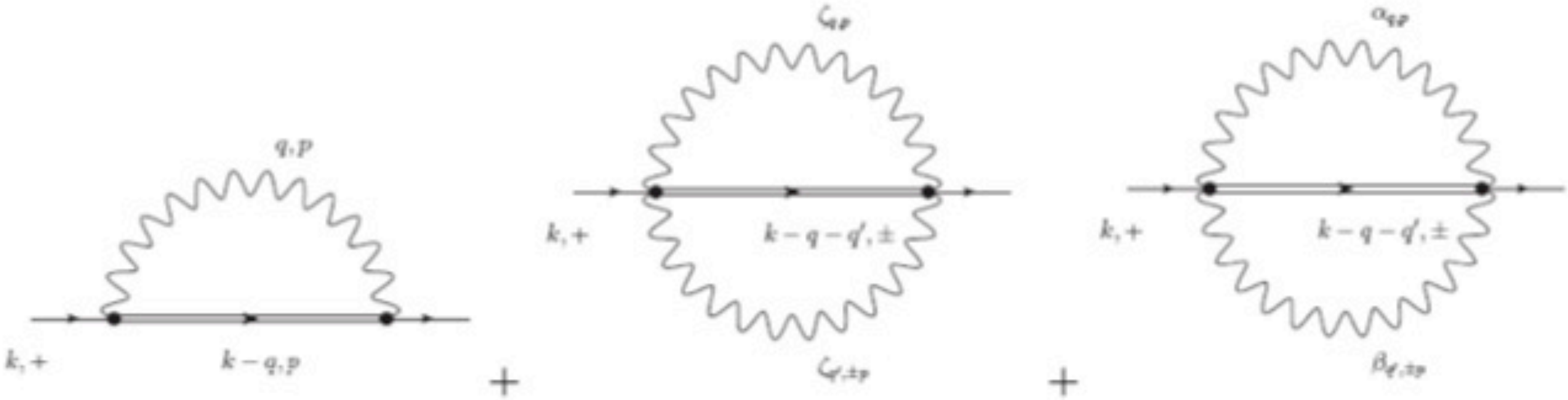
Feynman Diagrams

$\delta(\Pi^3_{i=1} p_i - 1)$



Self-consistent Born Approximation

$$\Sigma^{+}(k,\omega) \;=$$

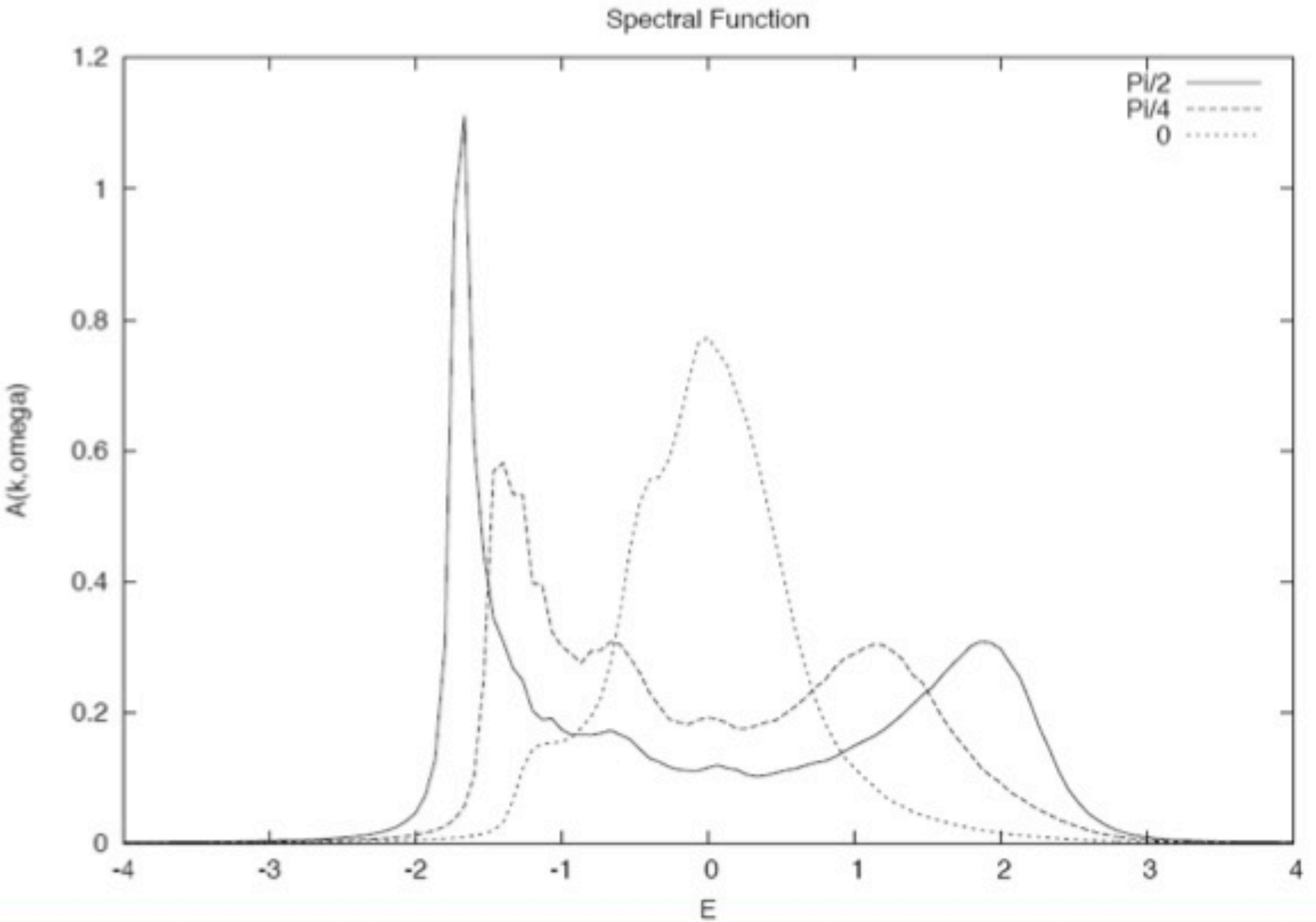


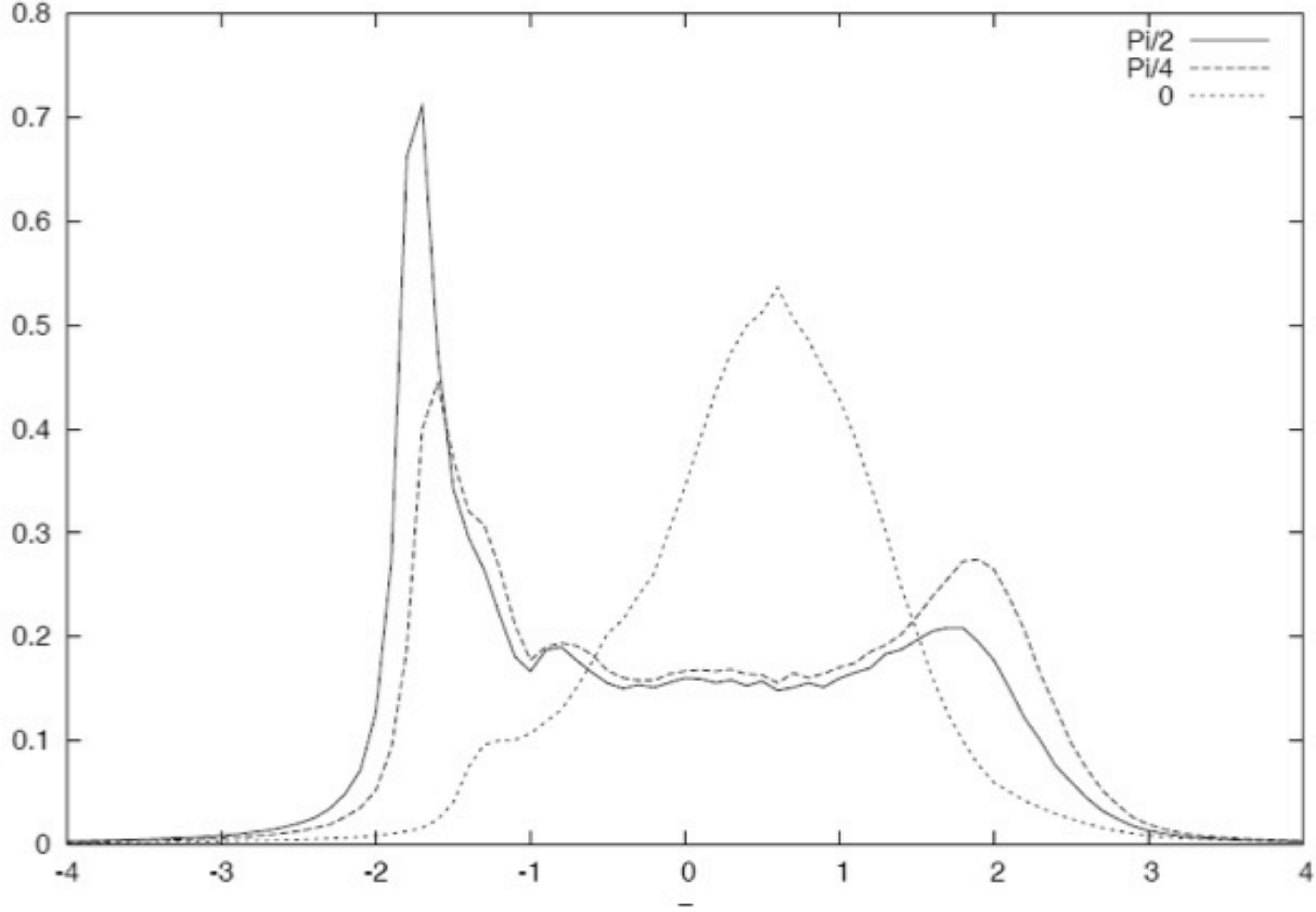
$$(\gamma_{k-q}\cosh\varphi_{q,p} + p\gamma_k \sinh\varphi_{q,p})^2$$

$$(\gamma_{k+q'}\cosh\varphi_{q,p_3}\sinh\varphi_{q',p_4} \pm \gamma_{k+q}\cosh\varphi_{q',p_4}\sinh\varphi_{q,p_3})^2$$

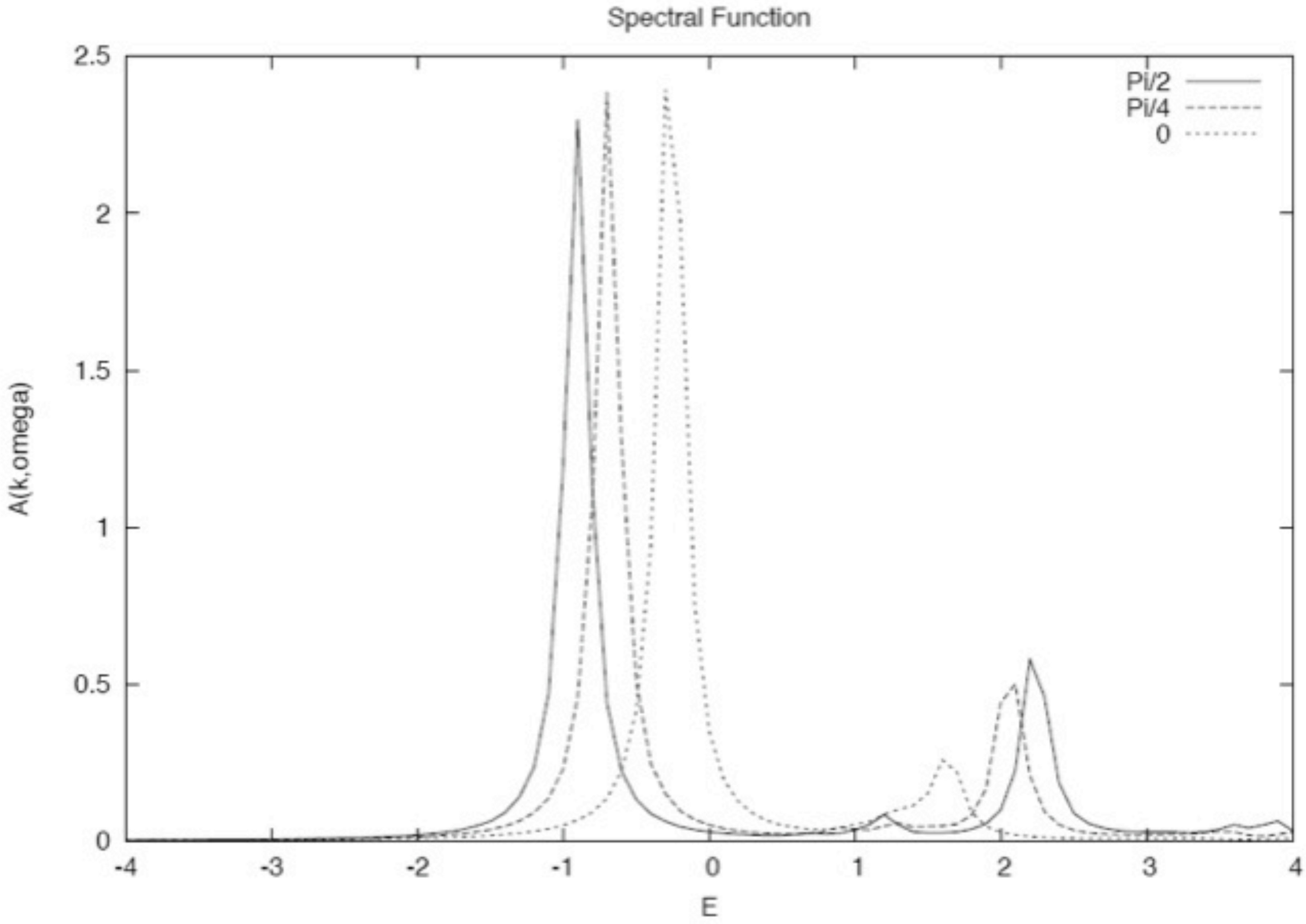
$$(\gamma_{k-q}\cosh\theta_{q,p}\sinh\theta_{q',\pm p} \pm \gamma_{k-q'}\cosh\theta_{q',\pm p}\sinh\theta_{q,p})^2$$

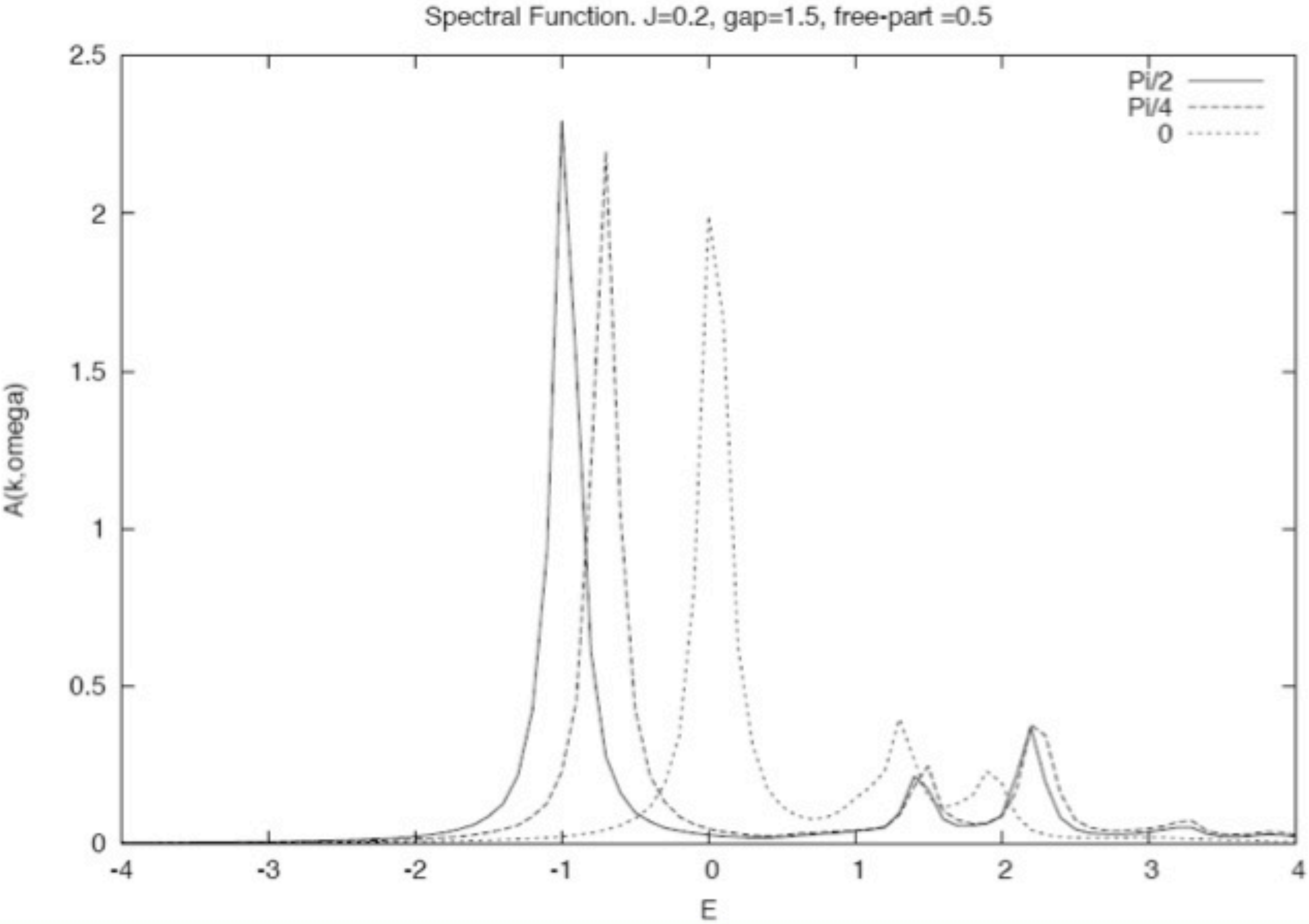
Simulation in the single layer case





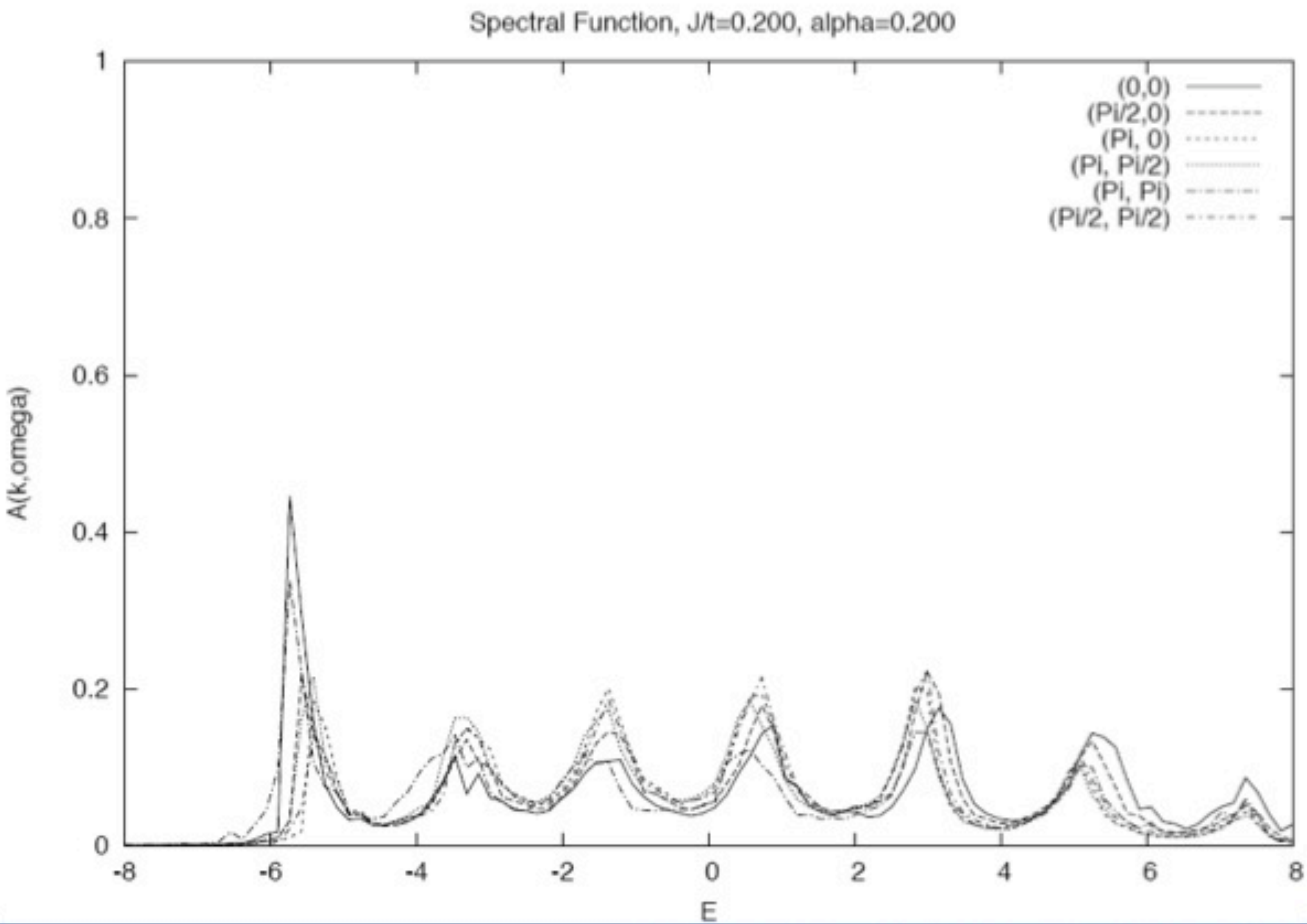
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
Pi/2
Pi/4
0

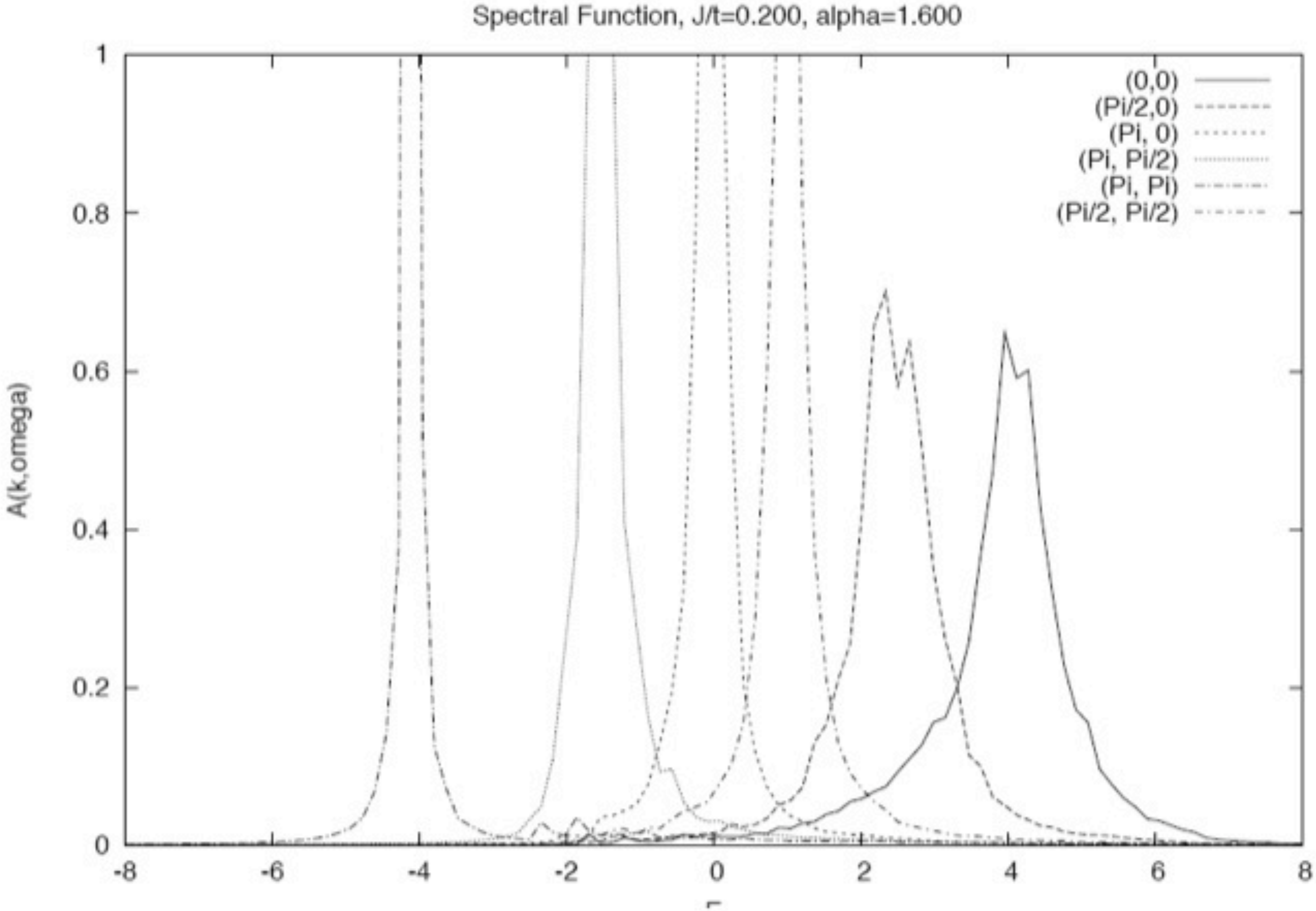

Spectral Function
Pi/2
Pi/4
0
A(k,omega)
E
2.5
2
1.5
1
0.5
0
-4
-3
-2
-1
0
1
2
3
4



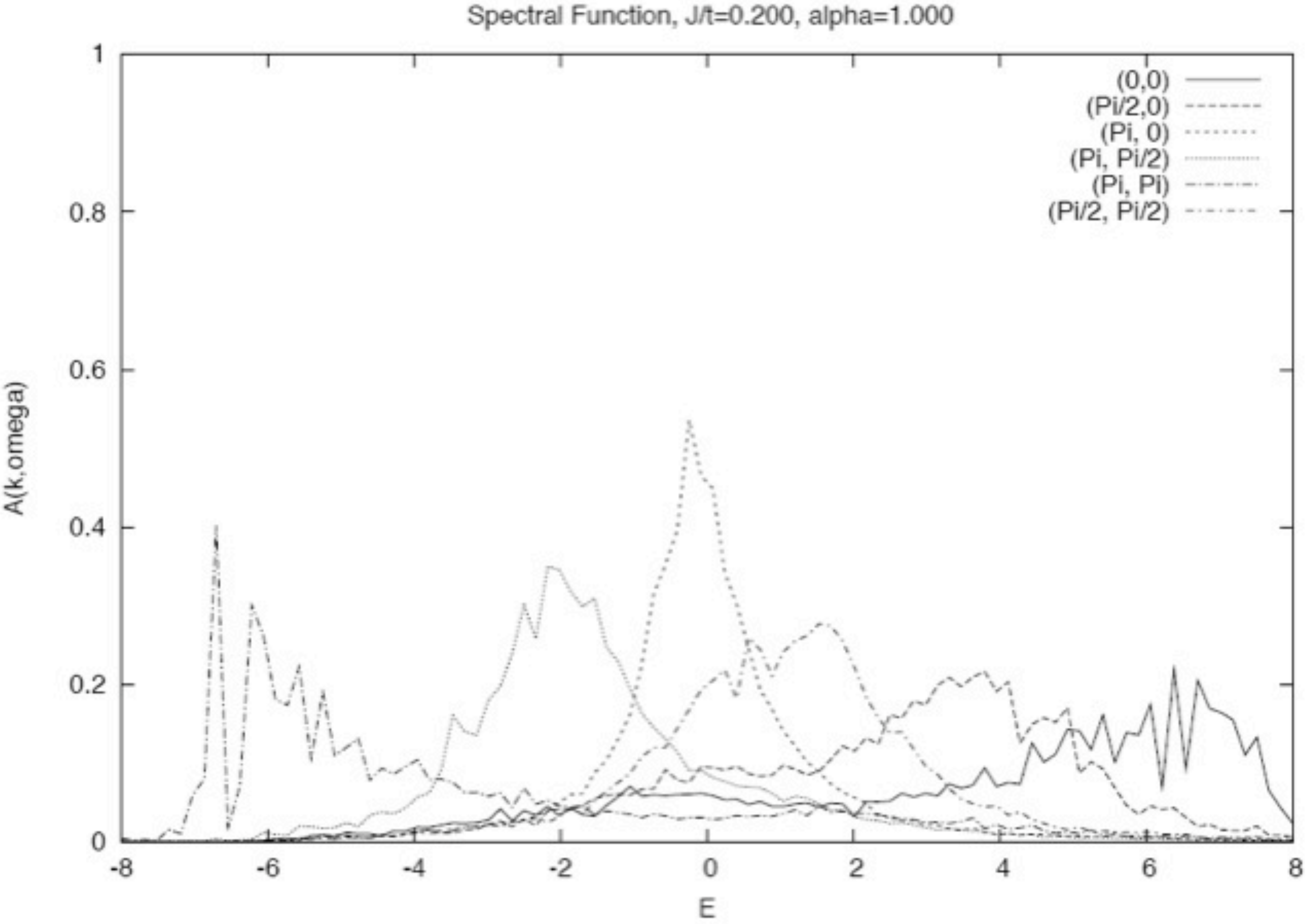
Spectral Function. J=0.2, gap=1.5, free-part =0.5
Pi/2
Pi/4
0
A(k,omega)
E
2.5
2
1.5
1
0.5
0
-4
-3
-2
-1
0
1
2
3
4

Bilayer numerical results





Spectral Function, J/t=0.200, alpha=1.600
A(k,omega)
(0,0)
(Pi/2,0)
(Pi, 0)
(Pi, Pi/2)
(Pi, Pi)
(Pi/2, Pi/2)
0
0.2
0.4
0.6
0.8
1
-8
-6
-4
-2
0
2
4
6
8


Spectral Function, J/t=0.200, alpha=1.000
A(k,omega)
E
(0,0)
(Pi/2,0)
(Pi, 0)
(Pi, Pi/2)
(Pi, Pi)
(Pi/2, Pi/2)
1
0.8
0.6
0.4
0.2
0
-8
-6
-4
-2
0
2
4
6
8

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Universiteit Leiden
The Netherlands