How a Neutral and Massless Superfluid can still exhibit Flux Quantization

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Quantization in Superfluids

A ring of superconducting material has the amazing property that the magnetic flux passing through it must be quantized. This effect has lead to a vast range of technological applications such as SQUIDs and RSFQ circuits.

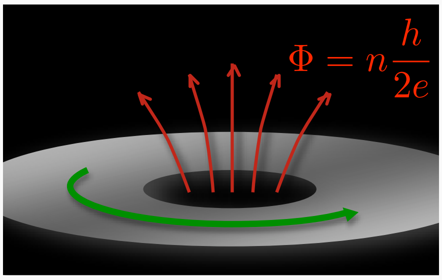

Figure 1: A circular supercurrent (in green) leads to a quantized magnetic flux (in red) through the ring.

Flux quantization is the direct result of the macroscopic coherence that is inherent to Bose-Einstein condensates. In all superfluid condensates the supercurrent is caused by a gradient in the macroscopic phase, $\vec{j} \sim \vec{\nabla}\phi$. In a circular loop the phase can only change with multiples of 2π, hence the **circular supercurrent is always quantized**. A circular electric current causes a magnetic flux, which is why superconductors display **magnetic flux quantization** (see Figure 1).

Flux quantization is closely related to the Meissner effect. If one cools a superconducting ring down to below the critical temperature in the presence of an external magnetic field, the flux inside the material is expelled by a screening current. When the external field is switched off, the screening currents remain causing a visible remnant quantized flux.

Charge nor Mass: Excitons

We are particularly interested in condensates of **excitons**, which are proposed to exist even at room-temperatures. An exciton is a bound state of an electron and a hole, and thus an exciton **carries no electrical charge**.

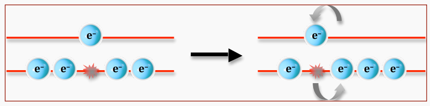

Figure 2: The hopping of an exciton does not lead to a transfer of mass.

Remember that the movement of a hole is equal to the movement of an electron in the opposite direction. Consequently, as can be seen in Figure 2, an exciton current **does not transfer mass**!

Without electric charge or mass a quantization effect would seem absurd. But is **quantization in exciton superfluids** really impossible?

Flux Quantization for Bilayer Excitons

Because an exciton is made out of a negative electron and a positive hole it has an **electrical dipole moment**. This is especially evident in bilayer exciton systems where the electrons and holes are bound to different layers (see Figure 3).

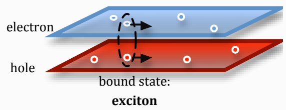

Figure 3: In bilayer excitons the electrons and holes live in different layers.

Correspondingly, a supercurrent of bilayer excitons can be described by **separate supercurrents of the electrons and holes** in the two layers. These two currents are equal in strength but flow in opposite directions.

One can put the two bilayers in the shape of two concentric cylinders as shown in Figure 4. The electron and hole currents cause flux quantization within both cylinders. As a result the **flux between the two layers is also quantized**. Since a moving electric dipole induces a relative weak magnetic field, the unit of quantization is reduced by the magnetic susceptibility χ_m :

$$\Phi = n\chi_m \frac{h}{e} \equiv n\chi_m \Phi_0$$

Ref: Rademaker, Zaanen and Hilgenkamp. Phys. Rev. B 83, 012504 (2011)

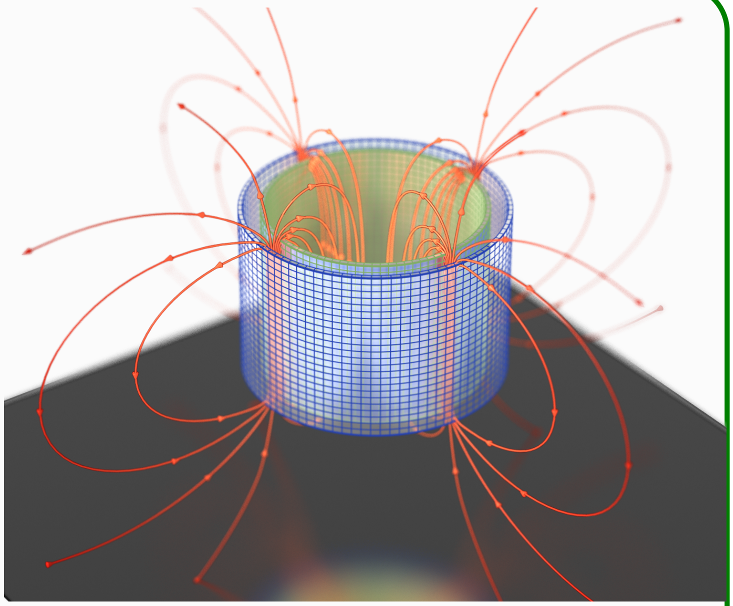

Figure 4: In concentric bilayers a supercurrent of excitons will exhibit a quantization of the flux in between the two cylinders. The magnetic field lines are shown in red. (Figure made by Jeroen Huijben.)

Experimental Realization

With a typical interlayer distance of 20 nm and a radius of 80 micron, one flux quantum corresponds to a field strength of 1 milli-Tesla. This indicates that with current technology it is possible to construct such concentric samples that enable us to measure the magnetic flux quantization with sufficient precision. Possible candidate systems to experimentally realize such concentric bilayers could be based on e.g., semiconductor 2DEGs, organic multi-layers, bilayer graphene or oxide heterostructures.

A bird's-eye view on the theory

The two-dimensional bilayer exciton condensate is described by an order parameter

$$\Delta(\vec{r}) = < c_n^\dagger(\vec{r}_n) c_p^\dagger(\vec{r}_p) > \equiv |\Delta(\vec{r})| e^{i\phi(\vec{r})},$$

where $c_n^\dagger(\vec{r}_n)$ creates an electron in the negative layer and $c_p^\dagger(\vec{r}_p)$ creates a hole in the positive layer. Given *U(1)* gauge invariance we can write out the free energy in terms of this order parameter. Consequently, the exciton current is

$$\vec{j} \equiv \frac{\hbar\rho}{m^*}\vec{\nabla}\phi = \frac{\rho e}{m^*}\vec{d} \times \vec{B}.$$

The free energy of a cylindrical sample as shown in Figure 4, with applied magnetic field B_z, equals

$$F[\Psi] \sim \oint d\theta \left[\frac{h}{e}\partial_\theta \phi - B_z 2\pi r d\right]^2$$

from which we can read off the induced quantized phase gradient, which in turn equals the dipole current. Using Maxwells equations we can infer the corresponding magnetic flux.

Finally, the flux quantization can be disturbed by either locally destroying the condensate (*phase slip)* or by defects in the material (*phase pinning*). Both effects limit the range of possible magnetic field strengths needed to see flux quantization.

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