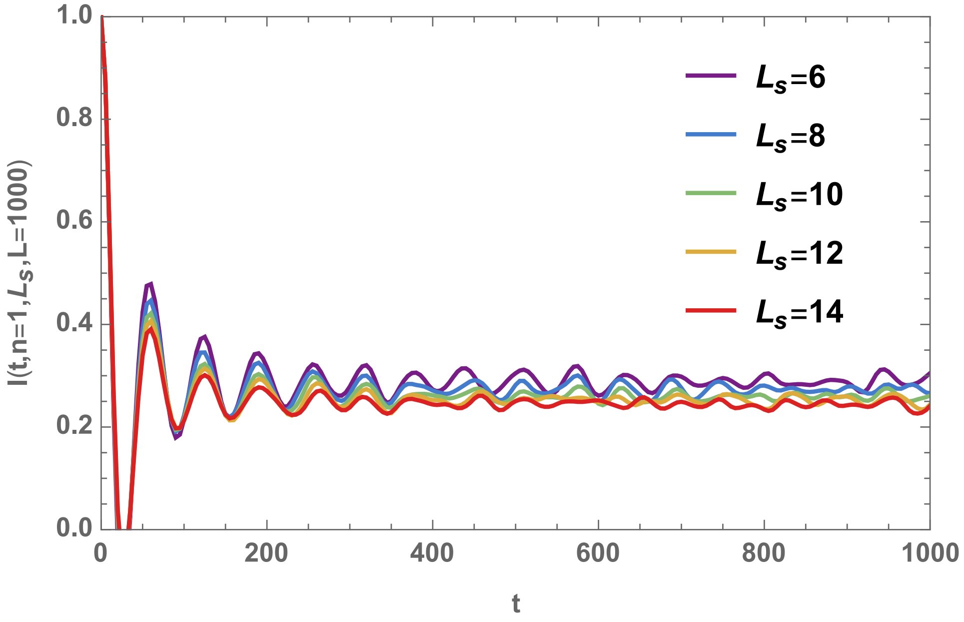
Unconventional Many-Body Localization in Long-Range Quantum Spin Glasses



Louk Rademaker, San Pedro del Pinatar, 28 August 2019

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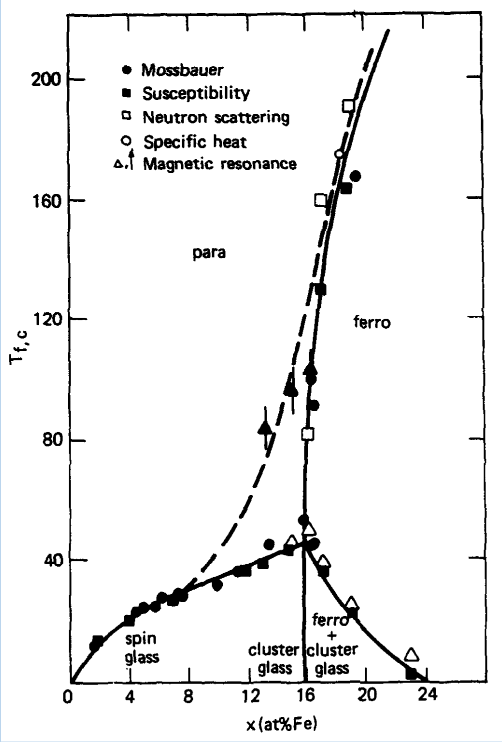
Overview

- **Introduce Spin Glass and MBL**
- **Our model: Long-Range Quantum Spin Glass**
  - Exact Diagonalization
  - Problem with long-range interactions
  - Entanglement
  - Long-range quenches
- **Possible experimental realization**

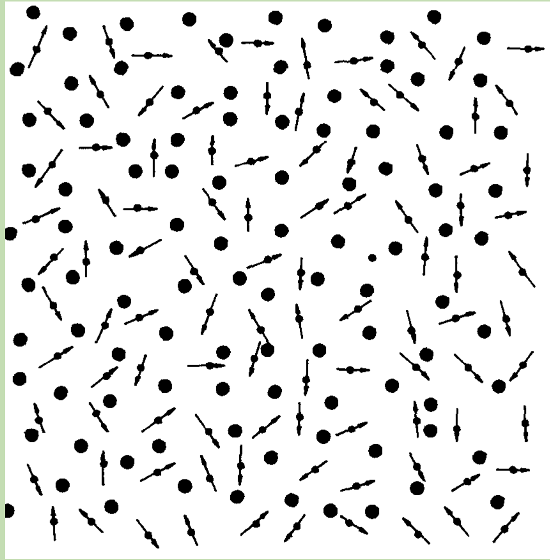
Work done with Dima Abanin , Andrés Somoza , and Miguel Ortuño 

Spin glass - Experiment

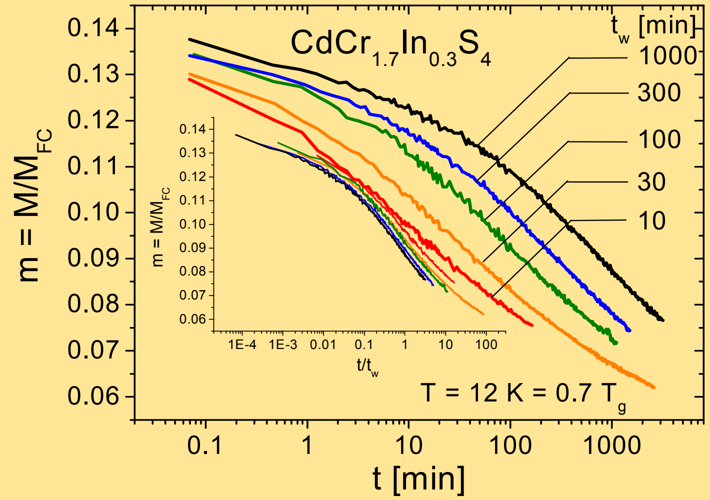
Randomly placed **magnetic impurities** leads to new low-T phase with:



Frozen spins but no magnetic long-range order



Dynamics **extremely slow** and system has '**memory**'



(thermo-remanent magnetization)

Spin glass - Theory

Spin model with **random** interactions and/or fields

Edwards-Anderson Model

$$H_{EA} = \sum_{\langle xy \rangle} J_{xy} \mathbf{m}_x \cdot \mathbf{m}_y$$

d=2,3

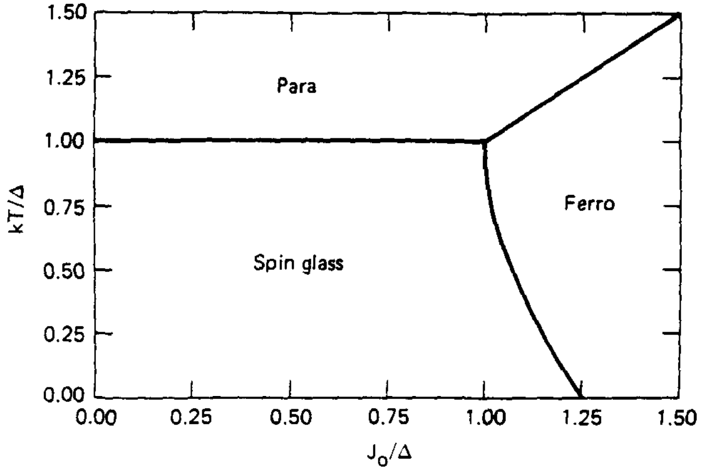
Sherrington-Kirkpatrick Model

$$H = \frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} J_{ij} Z_i Z_j$$

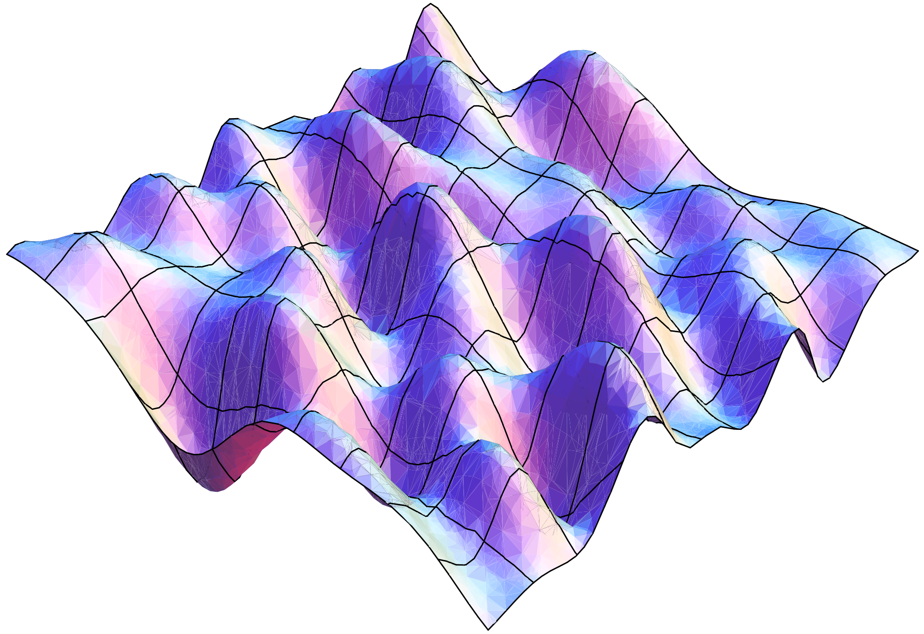
d=∞

Edwards-Anderson **order parameter**

$$m_{EA} = \frac{1}{N^2} \sum_{ij} |\langle \mathbf{m}_i \cdot \mathbf{m}_j \rangle|^2$$



Many-Body Localization



Quenched disorder leads to exponentially localized WF $\left|\Psi(r)\right| \sim e^{-r/\xi}$

Anderson localization Hamiltonian

$$H = -t\sum_{\langle ij\rangle}(c_i^\dagger c_j + h.c.) + \sum_i \mu_i n_i$$

can be **diagonalized** $H = \epsilon_i \tilde{n}_i$ **in d=1,2**

Local Integrals of Motion (LIOMs)

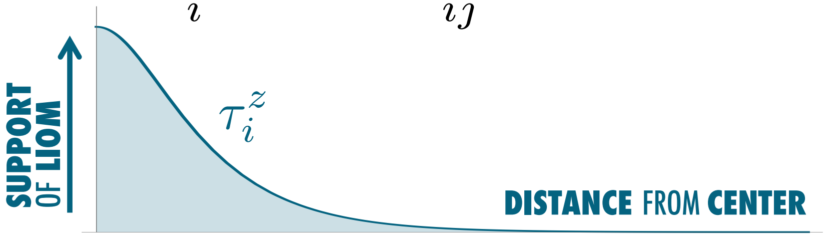
Question: What happens if you include **electron-electron interactions**?

Local Integrals of Motion (LIOMs)

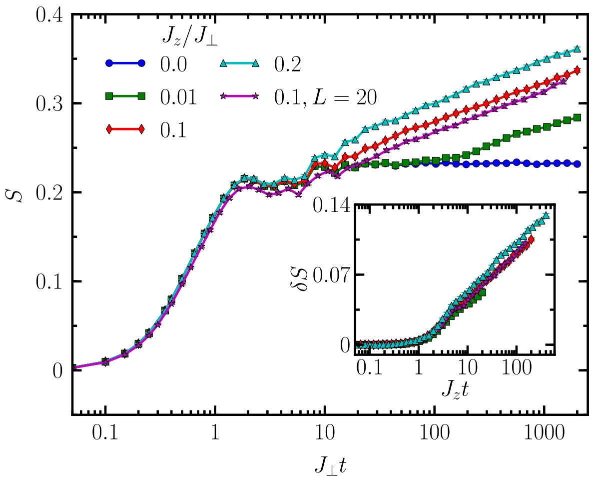
In d=1, local interactions ‘**dress**’ integrals of motion

Local Integrals of Motion (LIOMs)

$$H = \sum_\alpha \xi_\alpha n_\alpha + \frac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta \longrightarrow H = \sum_i \xi_i \tau_i^z + \sum_{ij} V_{ij}\tau_i^z\tau_j^z + \dots$$



In **Many-Body Localized phase** LIOMs still exist that **prevent thermalization**



Logarithmically slow growth of **entanglement/entropy**

Anderson ‘58
Basko, Aleiner, Altshuler ‘06
Huse, Nandkishore, Oganesyan ‘14
Bardarson, Pollmann, Moore ’12

Displacement Transformations

For every Hamiltonian, there exists a **unitary transformation** that brings it into the classical form

$$H = \sum_i \xi_i \tau_i^z + \sum_{ij} J_{ij} \tau_i^z \tau_j^z + \dots$$

Perturbation theory often fails due to resonances $c_\alpha \to c_\alpha + \frac{V_{\alpha\beta\gamma\delta}}{\xi_\alpha + \xi_\beta - \xi_\gamma - \xi_\delta} c_\beta^\dagger c_\gamma c_\delta$

Our solution: Consecutive application of exact 'small' unitaries

$$X = c_\alpha^\dagger c_\beta^\dagger c_\gamma c_\delta \qquad \tan 2\lambda = -\frac{V_{\alpha\beta\gamma\delta}}{\xi_\alpha + \xi_\beta - \xi_\gamma - \xi_\delta}$$

$$\mathcal{D}_\lambda(X) = \exp\left(\lambda(X^\dagger - X)\right)$$

Approximation: Cut off after a certain order in normal ordered operators

$$H = \sum_i \xi_i \tau_i^z + \sum_{ij} J_{ij} \tau_i^z \tau_j^z$$

$$\tau_i^z = \hat{U} \hat{n}_i \hat{U}^\dagger = \hat{n}_i + \alpha_{i;jk} \hat{c}_j^\dagger \hat{c}_k + \alpha_{i;jklm} \hat{c}_j^\dagger \hat{c}_k^\dagger \hat{c}_l \hat{c}_m$$

Compute J_{ij} and $\alpha_{i;jklm}$!

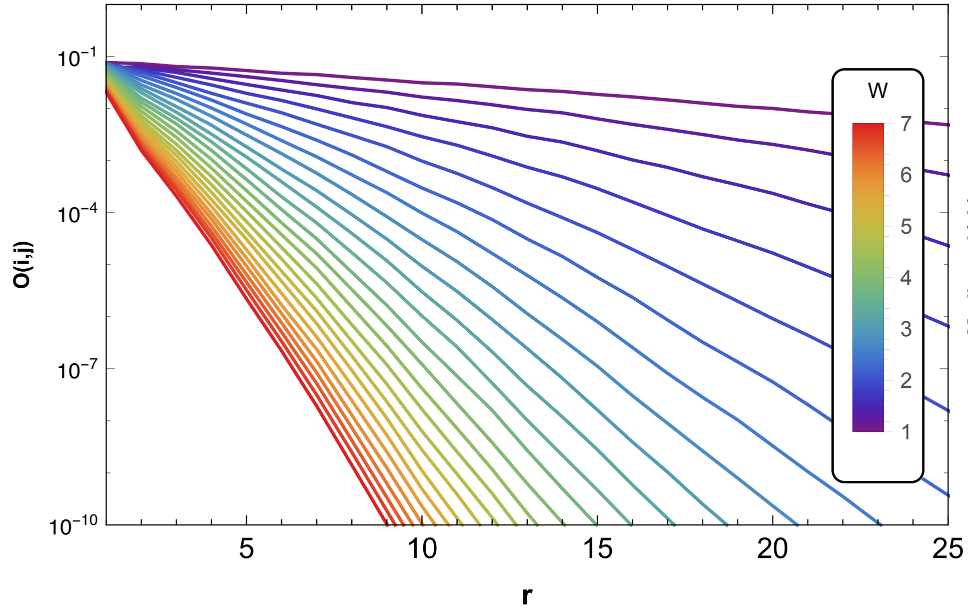
Some results

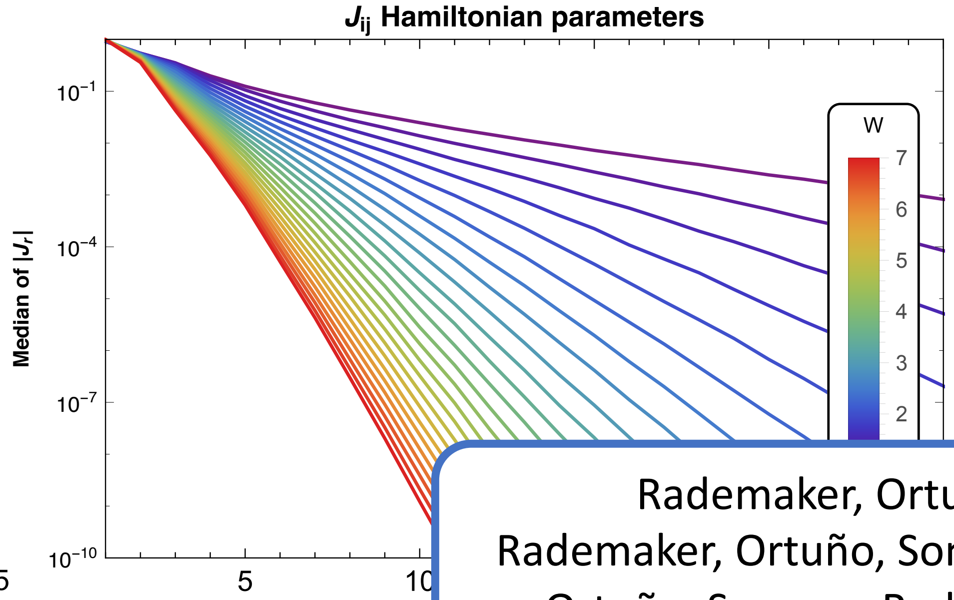
Our **model**: $H = \sum_{i=1}^{N} \epsilon_i n_i + t \sum_{i=1}^{N-1} (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + V \sum_{i=1}^{N-1} n_i n_{i+1}$ $\qquad \epsilon_i \in [-W/2, W/2]$

Take the **median** of all Integrals of Motion for many disorder realizations

As a function of **distance** and **disorder strength**

$$O(i,j) = \frac{\mathrm{Tr}\ \hat{\tau}_i^z \hat{n}_j}{\mathrm{Tr}\ \hat{n}_j}.$$





Rademaker, Ortuño PRL 2016
Rademaker, Ortuño, Somoza Ann Phys 2017
Ortuño, Somoza, Rademaker PRB 2019

Spin glass vs. Many-Body Localization

- Classical localized state of ‘**frozen**’ spins
- **Random** fields and/or interactions
- **High or infinite** dimensions, ór **long-range** interactions

Sherrington-Kirkpatrick Model

$$H = \frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} J_{ij} Z_i Z_j$$

- Quantum localized state of ‘**frozen**’ spins/electrons
- **Random** fields and/or interactions
- **One** dimension, ánd **short-range** interactions

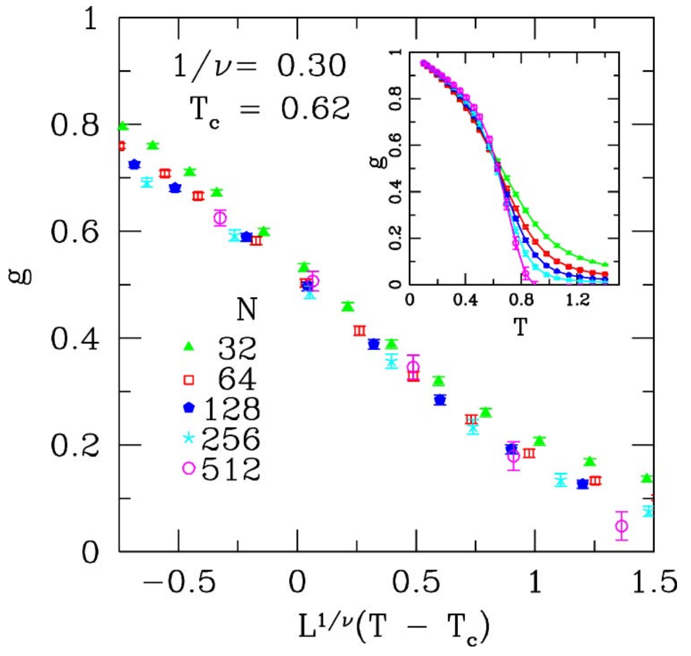
Random Field XXZ Chain

$$H = \sum_i J(X_i X_{i+1} + Y_i Y_{i+1} + \Delta Z_i Z_{i+1}) + h_i Z_i$$

Can you have ‘spin glass + MBL’?

$$H = \sum_{ij} \frac{J_{ij}}{|i-j|^{\alpha}} Z_i Z_j$$

- Take a long-range one-dimensional spin glass
- 0.5 < α < 1 to have spin glass order with random J
- Add a ***transverse field***

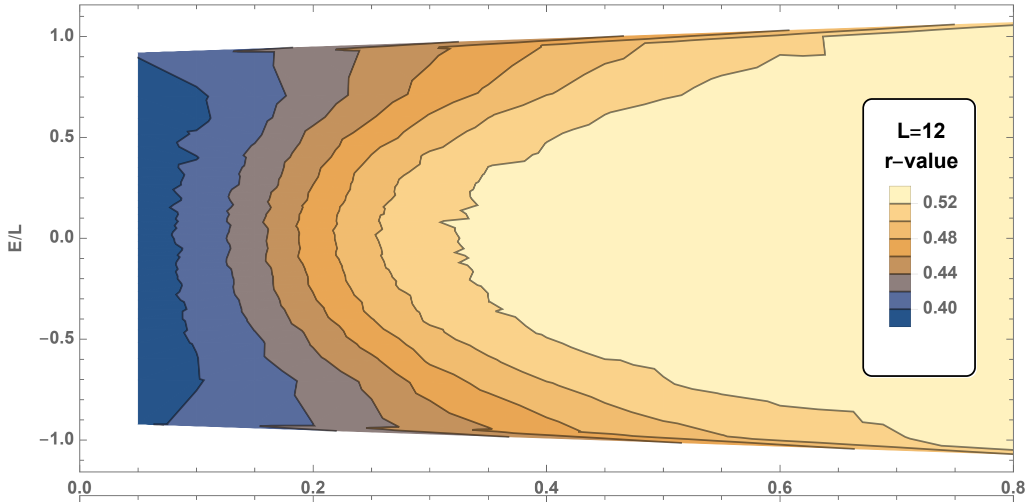


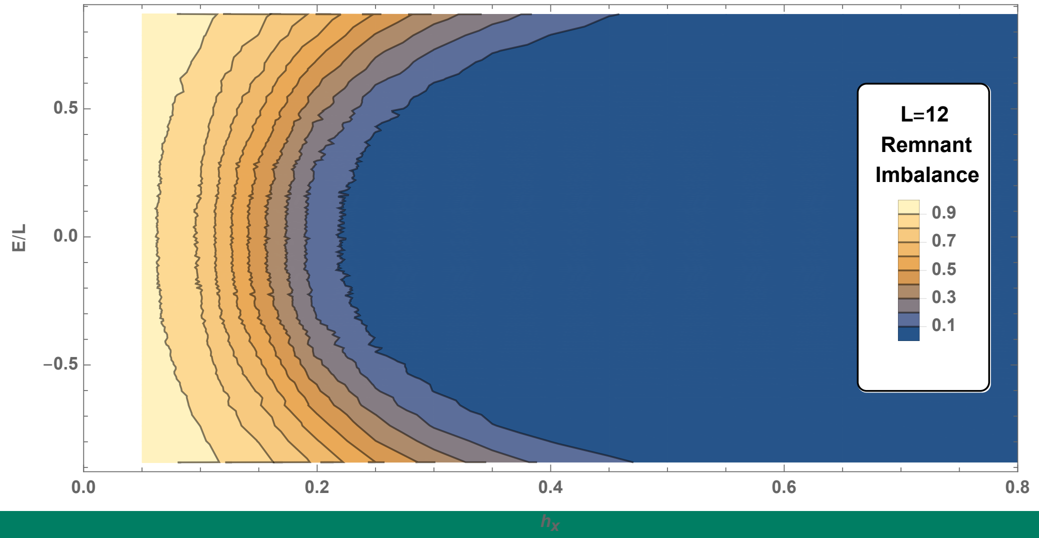
α=0.75

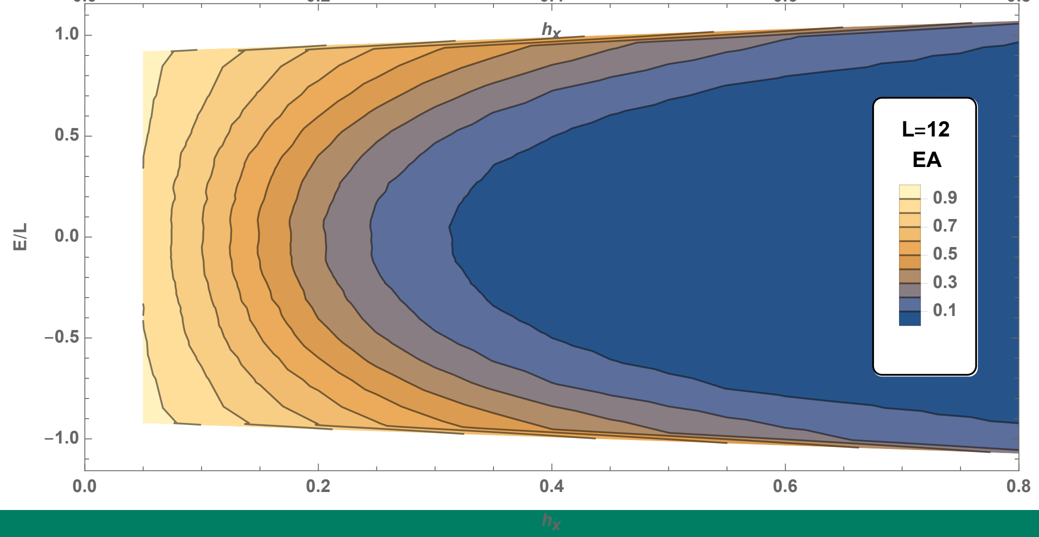
Ref: Katzgraber, Young, PRB 2003

Naïve Exact Diagonalization

- $r = \min(\delta_n, \delta_{n+1}) / \max(\delta_n, \delta_{n+1})$
- $m_{EA} = (\langle n | Z_1 Z_{L/2+1} | n \rangle)^2$
- $I = \lim_{t \to \infty} \frac{1}{L} \sum_i \langle Z_i(t) \rangle / \langle Z_i(0) \rangle$

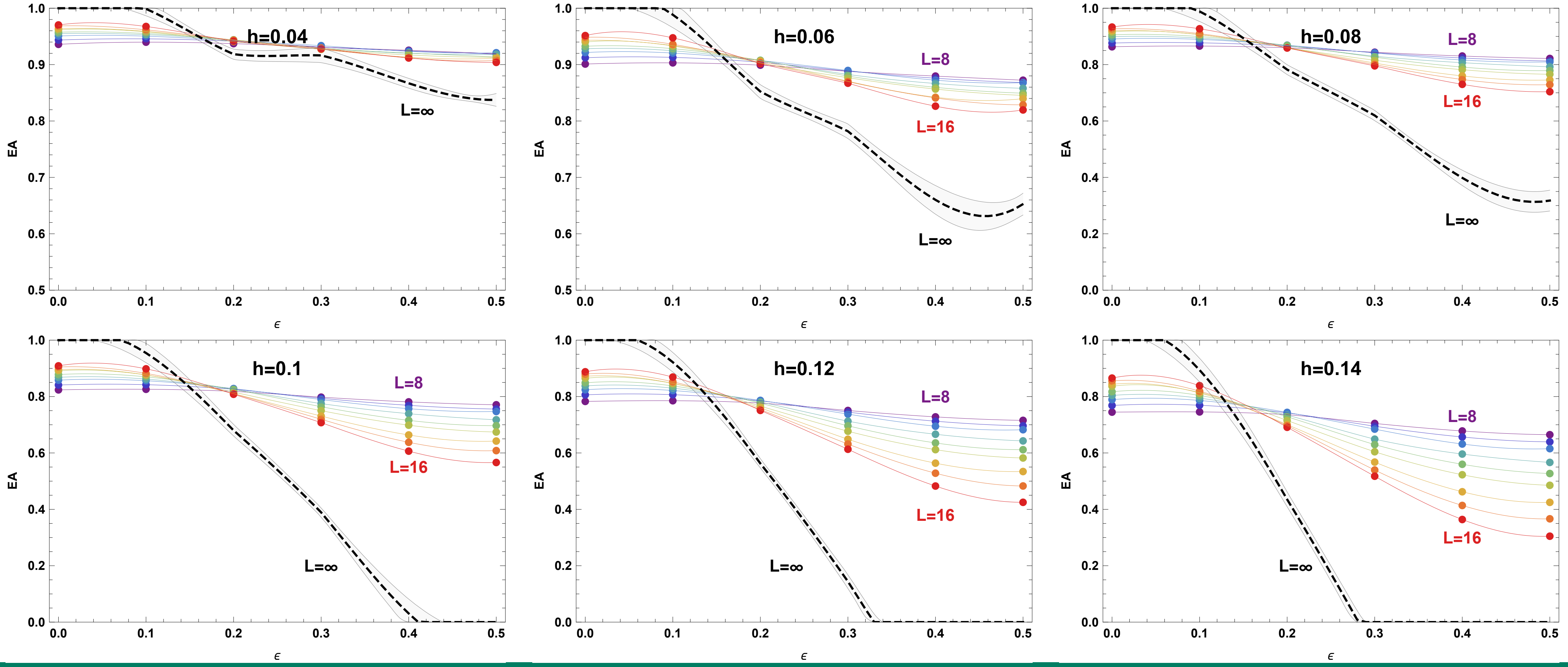






Finite size scaling of EA order parameter

$$m_{EA} = (\langle n|Z_1 Z_{L/2+1}|n\rangle)^2$$



Why MBL cannot stand Long-Range (1)

- Rewrite the interaction in terms of **'effective fields'**

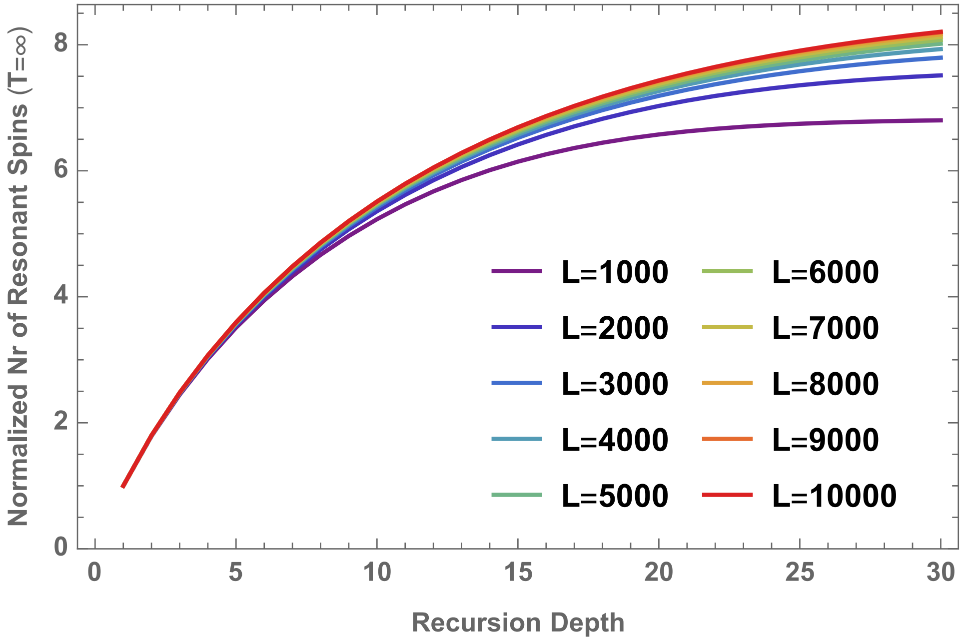
$$\sum_{ij} \frac{J_{ij}}{|i-j|^{\alpha}} Z_i Z_j = \sum_i Z_i \phi_i, \quad \phi_i \equiv \sum_j \frac{J_{ij}}{|i-j|^{\alpha}} Z_j$$

- Given any initial **spin glass** state, you can define '**resonant spins**' as having $|\phi_i| < h_x$

- For a random state **typical distance** between resonant spins is $d \sim h_x^{-1}$

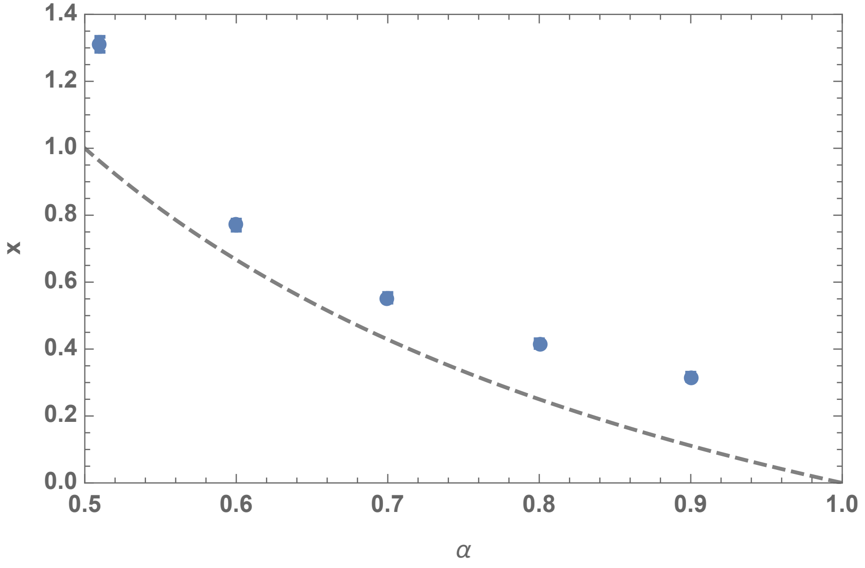
Why MBL cannot stand Long-Range (2)

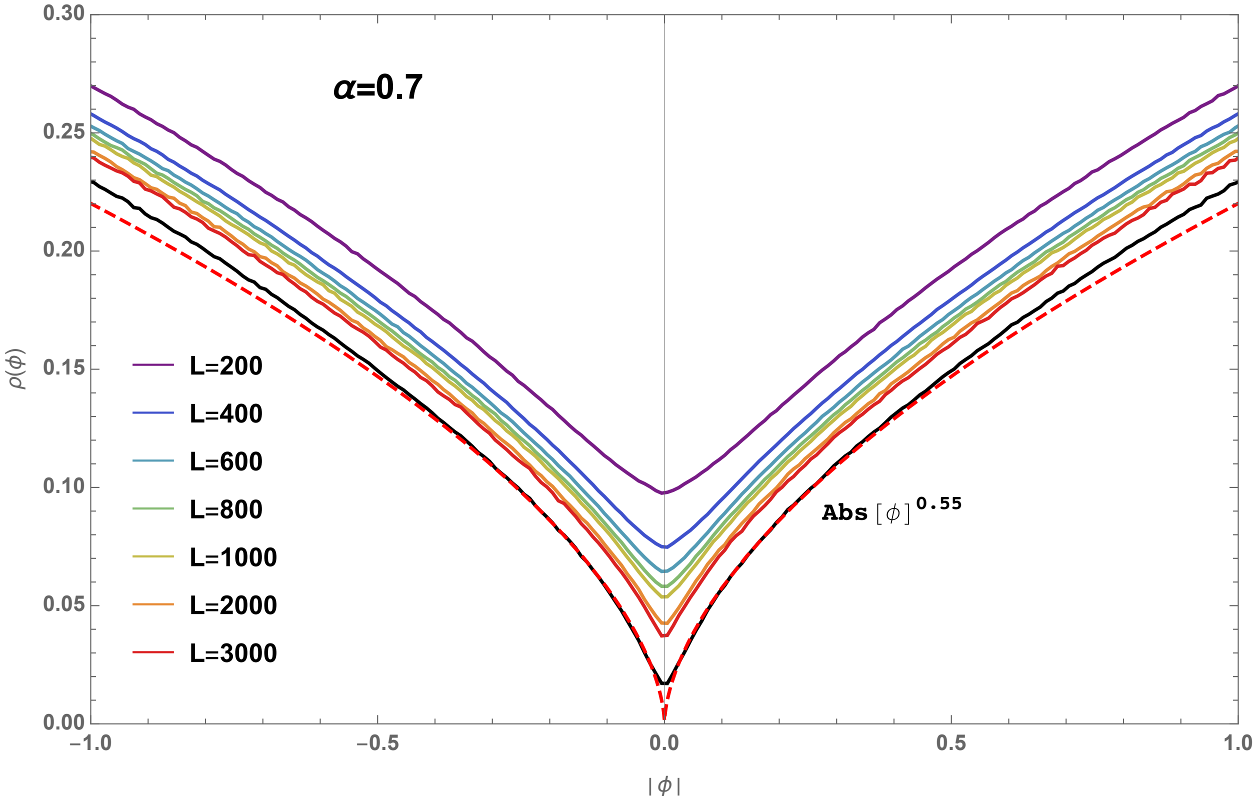
- Under dynamics, ‘resonant spins’ lose their polarization
- The effective field on other sites then typically changes $(\Delta\phi_j)^2 \sim \dfrac{1}{(2\alpha-1)d^{2\alpha}}$
- There is always a h_x such that $|\Delta\phi| \sim h_x^{\alpha} > h_x$, leads to **resonance avalanche**



The Way Out (1)

- At low T, the distribution of effective fields has a **soft gap** $\rho(\phi) \sim |\phi|^x$
- Typical distance $d \sim h_x^{-(x+1)}$
- Spectral **power *x*** depends on $\boldsymbol{\alpha}$





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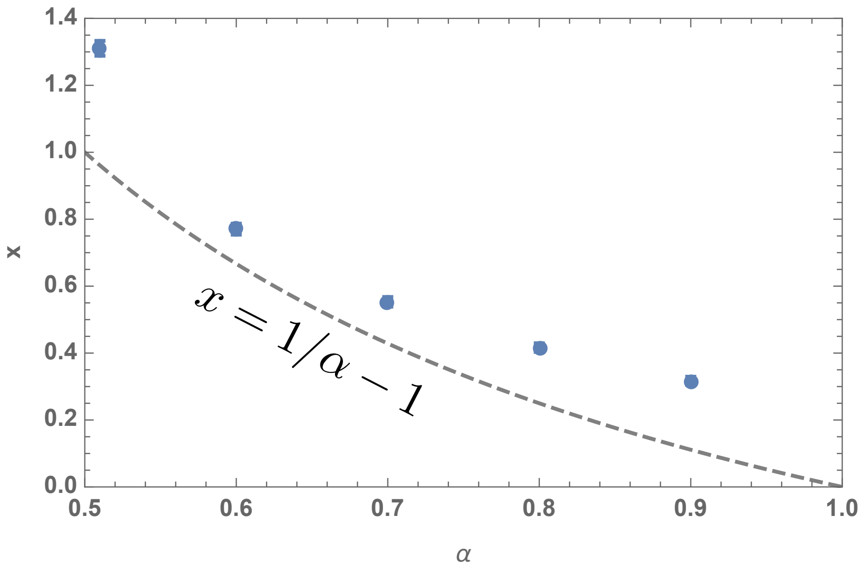
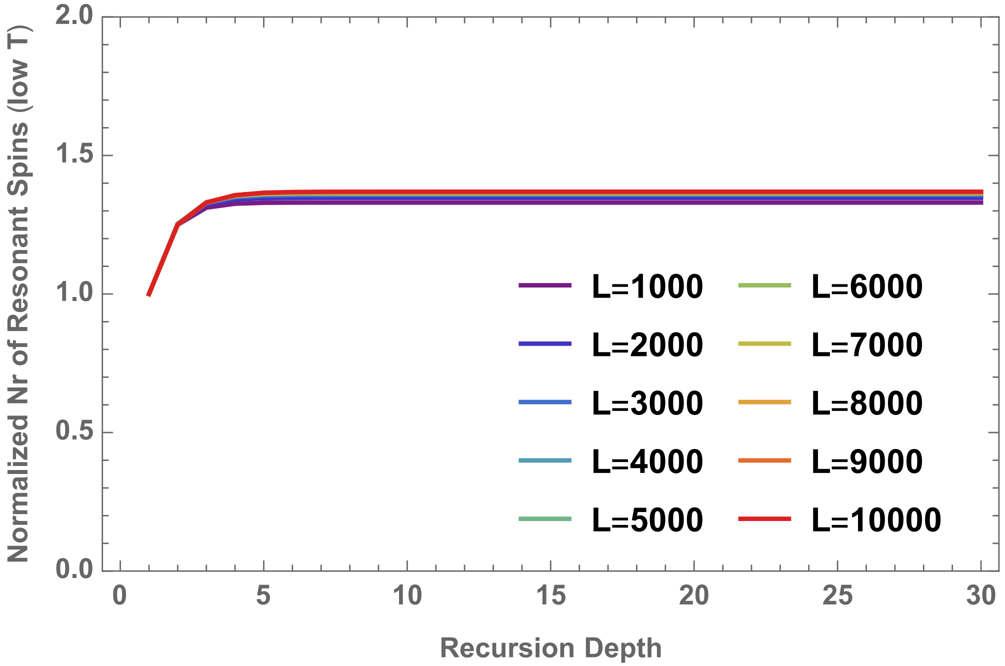
The Way Out (2)

- Resonant spins **losing polarization** leads to average **field change**

$$|\Delta\phi| \sim h_x^{\alpha(x+1)} < h_x$$

- As long as $x > 1/\alpha - 1$ there are **no resonance avalanches!**

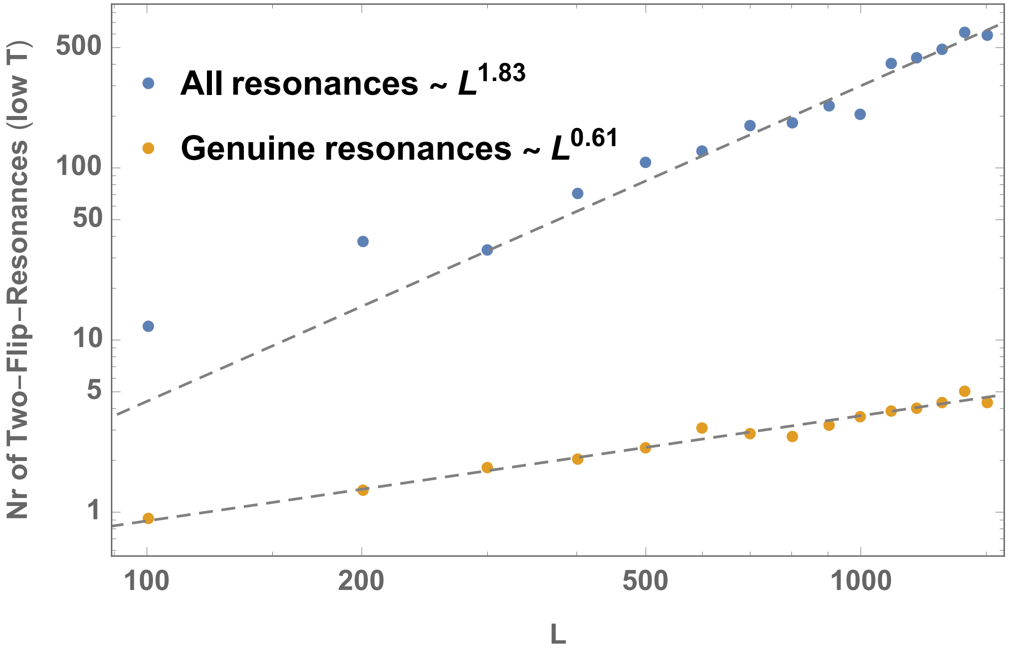


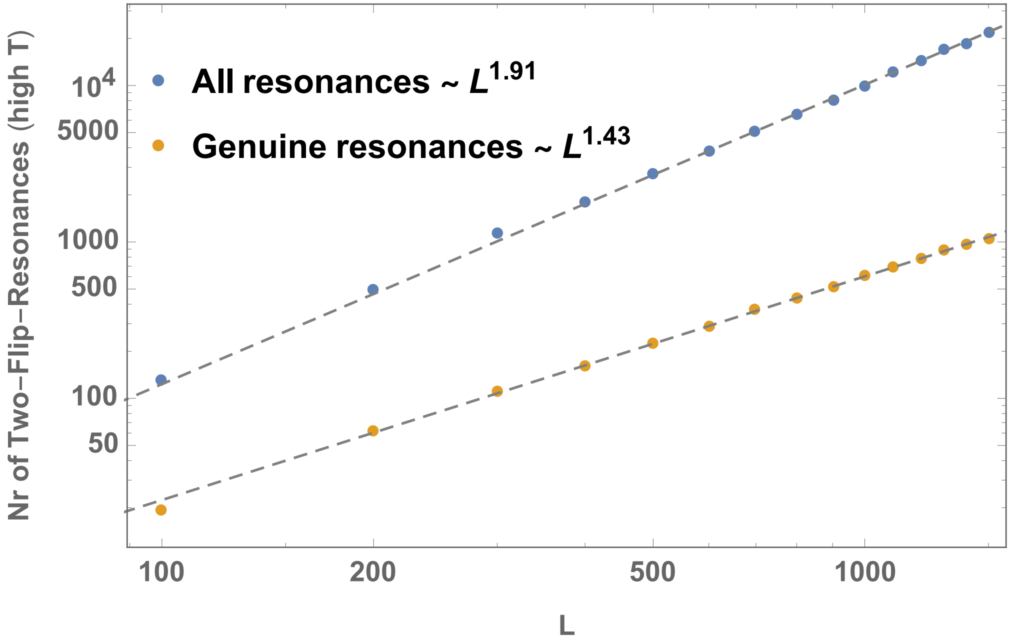


Two-spin-flip resonances

- What about **two-spin-flip** resonances?

$$A_{12\to\bar{1}\bar{2}} = \frac{h_x^2}{-\phi_1 - \phi_2 + 2\frac{J_{ij}}{r_{12}^\alpha}}\left(\frac{1}{-\phi_1} + \frac{1}{-\phi_2}\right)$$





Entanglement

- Let's look first at **entanglement** between **two resonant spins**

$$H_2 = \tilde{\phi}_1 Z_1 + \tilde{\phi}_2 Z_2 + \tilde{J_{12}} Z_1 Z_2 - h_x (X_1 + X_2)$$

- Maximum entanglement **entropy** generated is $S_E \sim \left(\frac{\tilde{J_{12}}}{h_x}\right)^2 \sim d^{-2\alpha}$
- Their interaction strength is $\tilde{J_{12}} = \frac{J_{r_1 r_2}}{|r_1 - r_2|^\alpha} \sim \frac{1}{d^\alpha}$ where d is **typical distance**

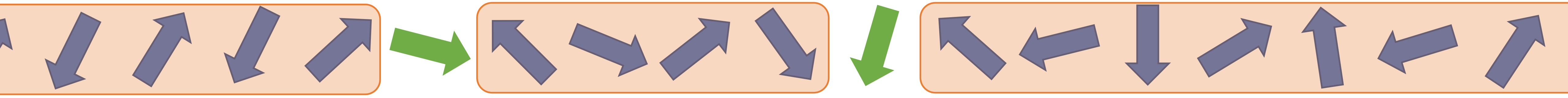
- **Approximate** bipartite many-particle entanglement as sum of two-spin entanglement

$$S_E \sim \sum_{i_L=-L/2}^{-1} \sum_{i_R=1}^{L/2} \frac{1}{|i_L - i_R|^{2\alpha}} \sim L^{2-2\alpha}$$

in between area law (L^0) and volume law (L^1)

Large-scale Quenches (1)

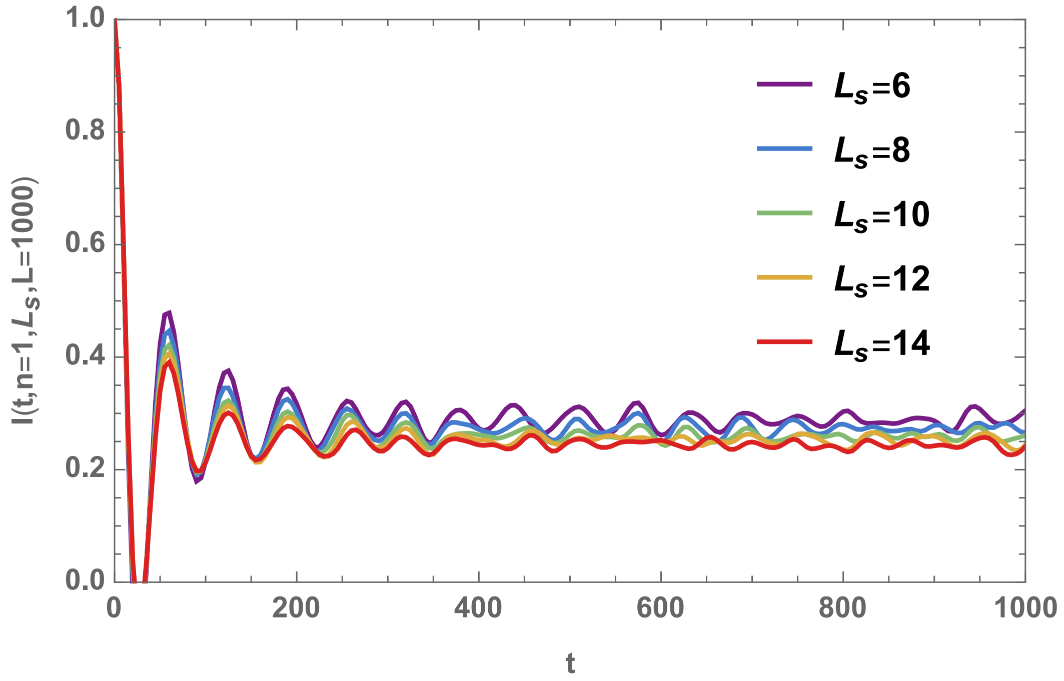
At low T / low energy density, the resonant spins are **scarce** but entangle
But: *Most* spins remain frozen, which allows **large-scale quench dynamics**!

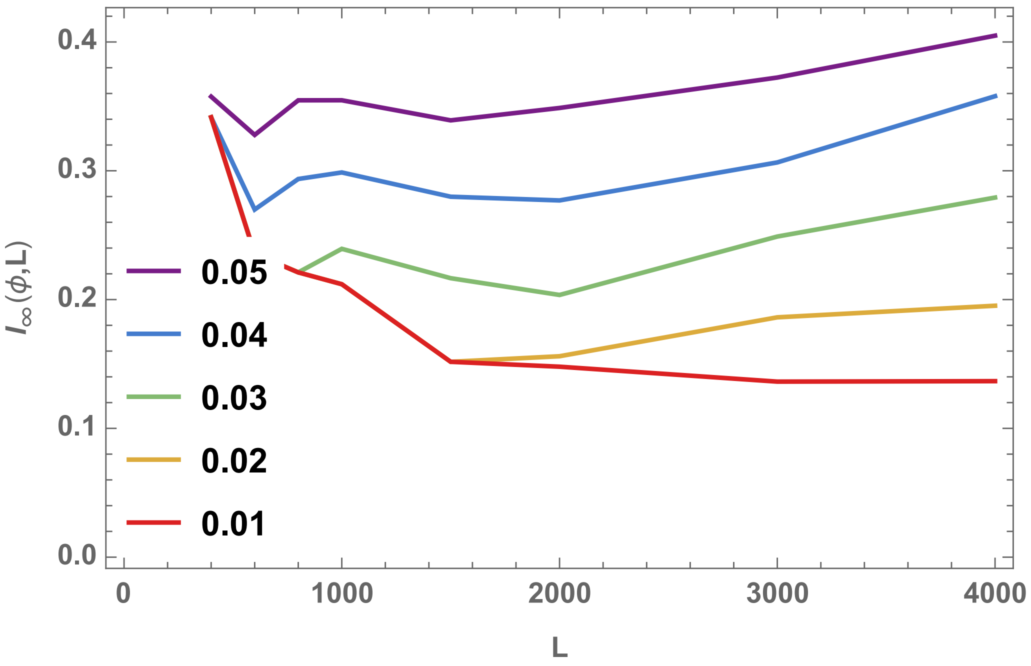


- From an initial state with 1000s of spins, identify L_s **few resonant spins**
- **Freeze** the remaining spins
- Do **Exact Diagonalization** on the resonant spins and **solve dynamics exactly**
- **Increase** L_s to get convergence

Large-scale Quenches (2)

Measure the **remnant imbalance** as a function of **initial effective field**





Shows that indeed the spin majority **remains frozen** but **resonant spins entangle**

Experimental realization

- Hyperfine states of Yb ions allows **exactly the right Hamiltonian**

nature physics | LETTERS

PUBLISHED ONLINE: 6 JUNE 2016 | DOI: 10.1038/NPHYS3783

Many-body localization in a quantum simulator with programmable random disorder

J. Smith[1]*, A. Lee[1], P. Richerme[2], B. Neyenhuis[1], P. W. Hess[1], P. Hauke[3,4], M. Heyl[3,4,5], D. A. Huse[6] and C. Monroe[1]

$$H_{\text{Ising}} = \sum_{i<j} J_{i,j}\sigma_i^x\sigma_j^x + \frac{B}{2}\sum_i \sigma_i^z + \sum_i \frac{D_i}{2}\sigma_i^z \qquad (1)$$

tunable long-range couplin
ally as $J_{i,j} \propto J_{\max}/|i-j|^{\alpha}$ (re
Hz). Here, we tune α betw
of the data $\alpha \approx 1.13$. We d

- **But** their experiment has **10 spins** only
  Typical distance between resonant spins is, for *h=0.05J*,

Infinite temperature	**Low temperature**
d = 60 sites	**d = 100 - 500 sites**

Summary / Outlook

$$H = \sum_{ij} \frac{J_{ij}}{|i-j|^\alpha} Z_i Z_j - h_x \sum_i X_i$$

- Long-range interaction model has phase with **most** spins frozen at low T
- Resonant spins are **scarce** but nonetheless interact, can study large-scale quenches
- Bipartite entanglement $S_E(L) \sim L^{2-2\alpha}$
- *Questions*: Many-particle transitions between different "valleys"?
  Relation to glassiness and the landscape problem?