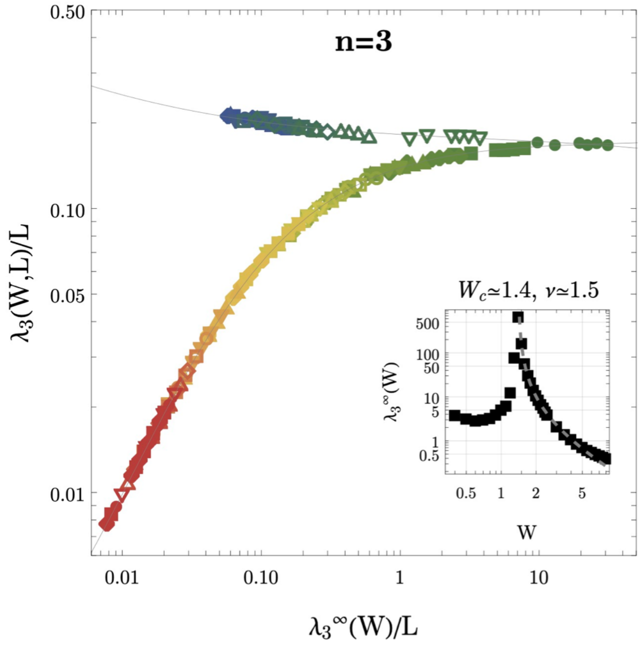


Scaling Theory of Few-Body Delocalization

Louk Rademaker

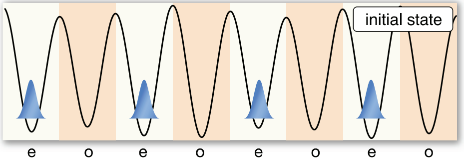
Wednesday 24 August 2022, Manchester

Many-Body Localization: Phenomenology

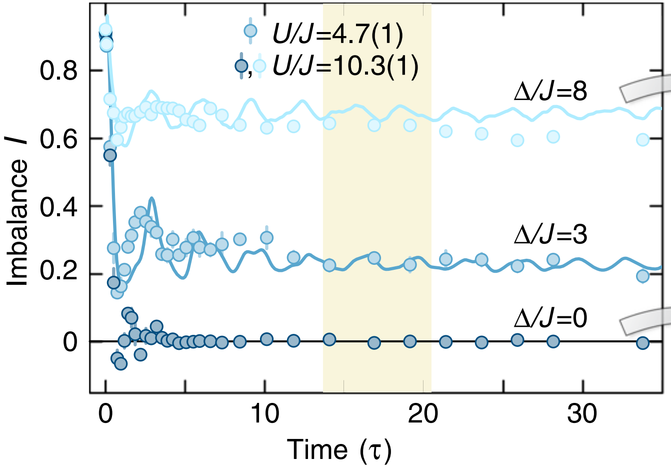
Example: **local charge density** after a **quantum quench** in cold atom chain

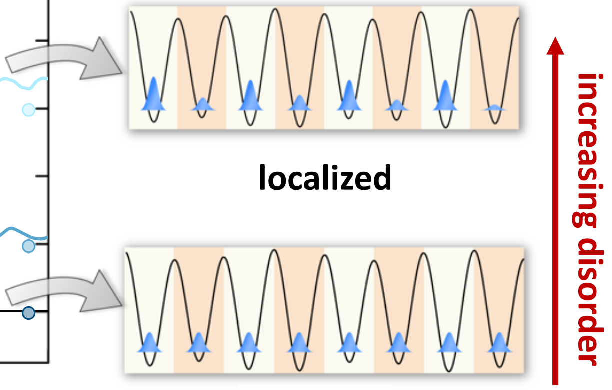
$$\hat{H} = -J \sum_{\langle ij \rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} + \sum_{i\sigma} \Delta_i n_{i\sigma} + U \sum_i n_{i\uparrow} n_{j\downarrow}$$

hopping **disorder** **interactions**



initial state





Ref: Schreiber, Science 2015

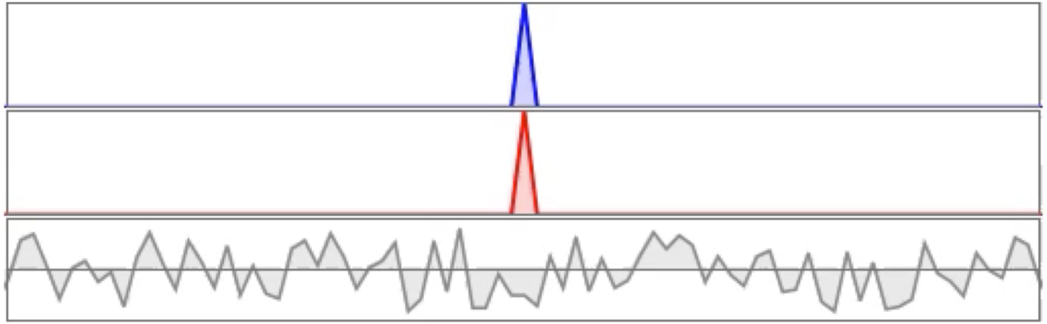
Anderson Localization

Disorder without **interactions**:

$$H = -t\sum_{\langle ij\rangle}(c_i^\dagger c_j + h.c.) + \sum_i \mu_i n_i$$

hopping **disorder**

Ref: Anderson '58



No disorder: ballistic spread of wavefunction

Disorder: localization

Disorder potential

In d=1 or d=2 dimensions all wavefunctions are **exponentially localized:** $|\Psi(r)| \sim e^{-r/\xi}$

Occupation number of each wavefunction is a **Local Integral of Motion**: $H = \epsilon_i \tilde{n}_i$

Many-Body Integrals of Motion

Add **interactions** to the Anderson insulator, still **get LIOMs?**

$$H = \sum_i \epsilon_i n_i + \frac{1}{2}\sum_{ijkl} V_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

Anderson LIOMs **interactions**

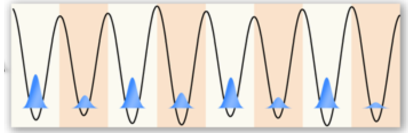
'classical' Hamiltonian

$$H = \sum_i \epsilon_i \tilde{n}_i + \sum_{ij} V_{ij}\tilde{n}_i\tilde{n}_j + \ldots$$

Yes! Dress the Anderson LIOMs: $\tilde{n}_i = U n_i U^\dagger = n_i + \alpha_{i;jklm} c_j^\dagger c_k^\dagger c_l c_m + \ldots$

Serbyn et al, PRL 2013; Huse et al, PRB 2014

In **d=1** with **short-range** interactions and **strong disorder**: **MBL**



localized

Localized $\mathrm{Tr}\left[\tilde{n}_i\, n_j\right] \sim e^{-|r_i - r_j|/\xi}$

Short-range interactions $V_{ij} \sim e^{-|r_i - r_j|/\xi_V}$

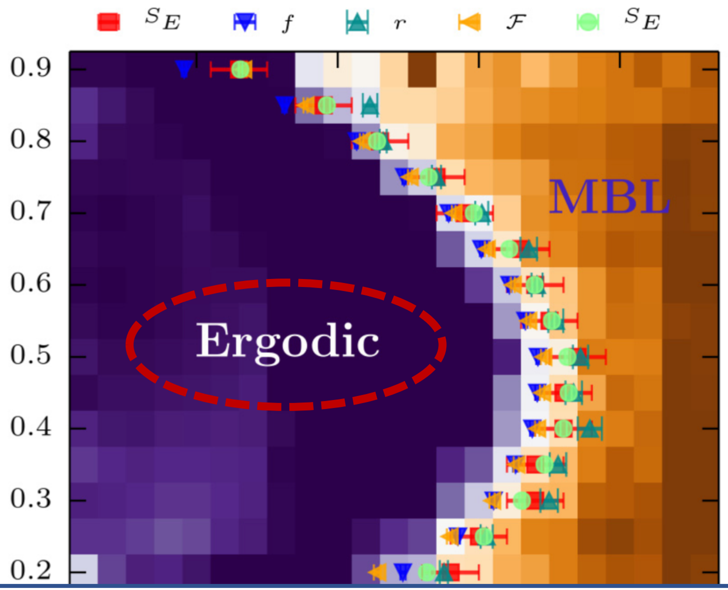
Ref: **Rademaker**, Ortuño, PRL 2016

Many-Body Delocalization?

Ergodic phase is **weird**!

Many-body excitations are **nonlocal** but single-particle states are **local**

Breakdown of **adiabatic** principle!



MBL =
"**Dressed**" Anderson insulator

Many-body state has **same excitations** as single-particle spectrum

Compare:
Fermi liquid
Magnetically ordered states

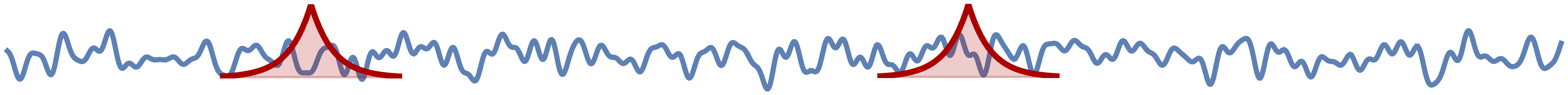
How to make a **delocalized** many-body state out of **localized** **particles**?

Ref: Luitz PRB 2015

Few-Body States

Short-range interactions $$H = \sum_{i=1}^{N} \epsilon_i n_i + t \sum_{i=1}^{N-1} (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + V \sum_{i=1}^{N-1} n_i n_{i+1}$$

What are the possible states of **n=2** or **3 particles**?



When particles are **far apart**: *n*-particle state is **unaffected**

When particles are **close**: **changed** *n*-particle states!

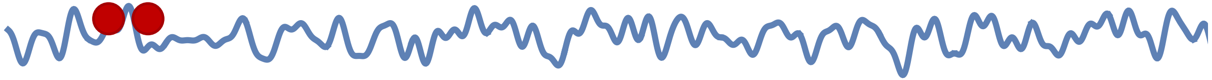
Expectation: seeds of many-body delocalization

Few-Body Greens Functions

How to **quantify** this? **Greens** functions!

One-particle: $G_1(x; y; E) = \langle 0 | c_x (E - H)^{-1} c_y^\dagger | 0 \rangle$

Two-particle: $G_2(x_1, x_2; y_1, y_2; E) = \langle 0 | c_{x_2} c_{x_1} (E - H)^{-1} c_{y_1}^\dagger c_{y_2}^\dagger | 0 \rangle$



Greens function allows for effective **localization length**

One-particle: $\lambda_1^{-1}(W, L) = -\frac{2}{L-1} \langle \log |G_1(1; L)|^2 \rangle_{\mathrm{dis}}$

Two-particle: $\lambda_2^{-1}(W, L) = -\frac{2}{L-2} \langle \log |G_2(1, 2; L-1, L)|^2 \rangle_{\mathrm{dis}}$

Which is related to the **transmission coefficient** $T_n(W, L) = \exp\left(\frac{-2L}{\lambda_n(W, L)}\right)$

Scaling theory

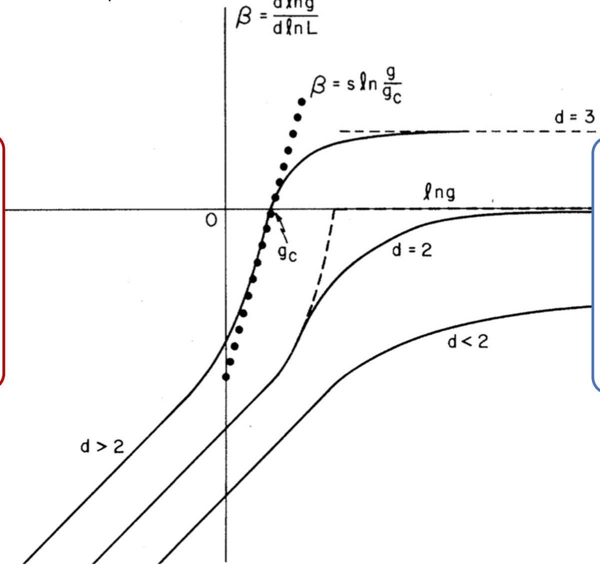
Dimensionless **conductance** $g(L)$ is a general form of transmission coefficient

Determined by **scaling** $\beta(g) = d\ln g / d\ln L$

small *g: localization*

$$g = g_0 e^{-L/\xi}$$

$$\beta(g) \to \ln g/g_0$$



large *g: bulk conduction*

$$g = \sigma L^{d-2}$$

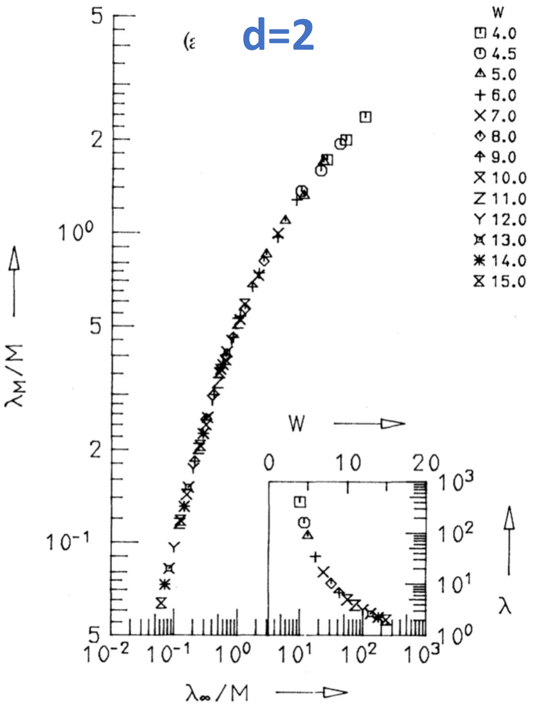
$$\beta(g) \to d - 2$$

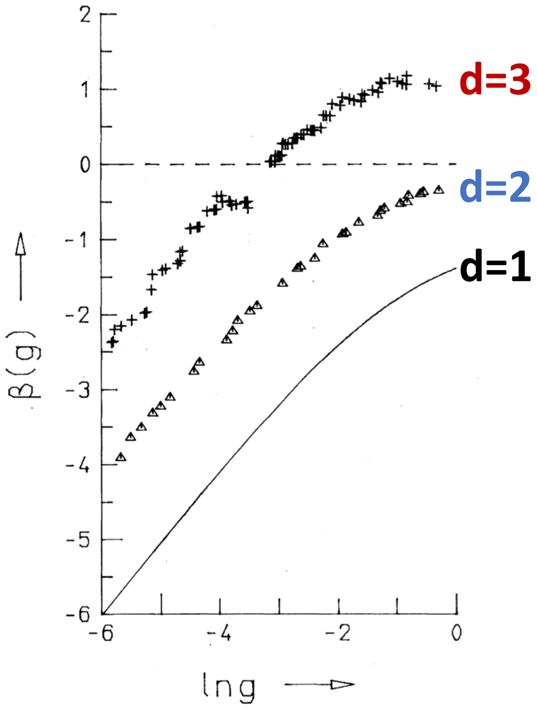
Ref: Abrahams PRL 1979

Numerical results for scaling theory

$$\lambda^{-1} = \lim_{N \to \infty} [2(N-1)]^{-1} \ln \mathrm{Tr} |\langle 1|G(N)|N\rangle|^2$$

$$\lambda(W, M)/M = f_d(\lambda_\infty(W)/M)$$





Ref: MacKinnon PRL 1981

Calculating few-body Greens functions

Exact calculation of Greens function is **inefficient** so use a **trick**

Two-particle **noninteracting** Greens function is

$$G_2^{(0)} = \sum_{mn} \frac{\phi_{x_2 n}\phi_{x_1 m}\phi_{y_1 m}\phi_{y_2 n} - \phi_{x_2 m}\phi_{x_1 n}\phi_{y_1 m}\phi_{y_2 n}}{E - \epsilon_m - \epsilon_n}$$

Dyson's equation states $G_2 = G_2^{(0)} + G_2^{(0)} H_{\text{int}} G_2$

But **local interactions** only act on $\mathcal{O}(L)$ part of Hilbert space

Calculate the **restricted Greens function** $\tilde{G}_2 = \tilde{G}_2^{(0)} + \tilde{G}_2^{(0)} H_{\text{int}} \tilde{G}_2$

Speeds up the computation of localization length $\mathcal{O}(L^6) \rightarrow \mathcal{O}(L^4)$

Ref: Van Oppen PRL 1996; Ortuno EL 1999

Scaling of two-body states in d=2

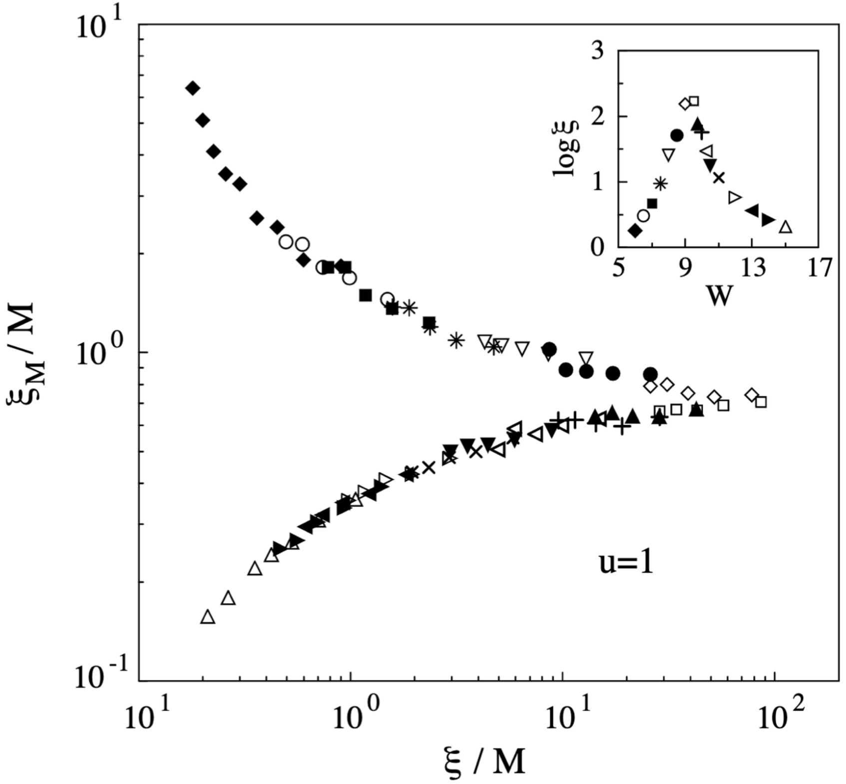
2d **Bose-Hubbard**

$$H = t \sum_{\{i,k\},j} |i,j\rangle\langle k,j| + t \sum_{i,\{j,l\}} |i,j\rangle\langle i,l|$$

$$+ \sum_{i,j} |i,j\rangle(\epsilon_i + \epsilon_j)\langle i,j| + U \equiv H_0 + U$$

Two-body states

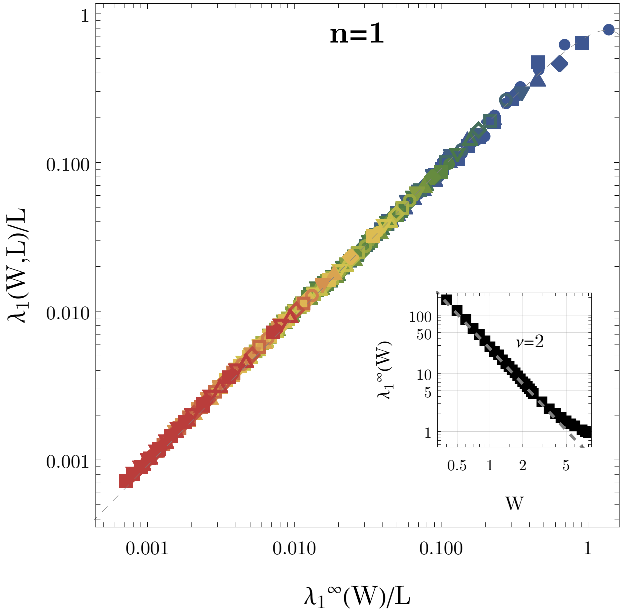
$$\ln \mathrm{Tr}\, |\widetilde{G}(l)|^2 \equiv \left\langle \ln \sum_{i,j} |\widetilde{G}(1,i;l,j)|^2 \right\rangle$$



Ref: Ortuno EL 1999

Scaling of few-body states in d=1

Scaling function $\lambda_n(W,L)/L = f_n^{\pm}(\lambda_n^{\infty}(W)/L)$



Ref: **Rademaker** PRB 2021

Beta function

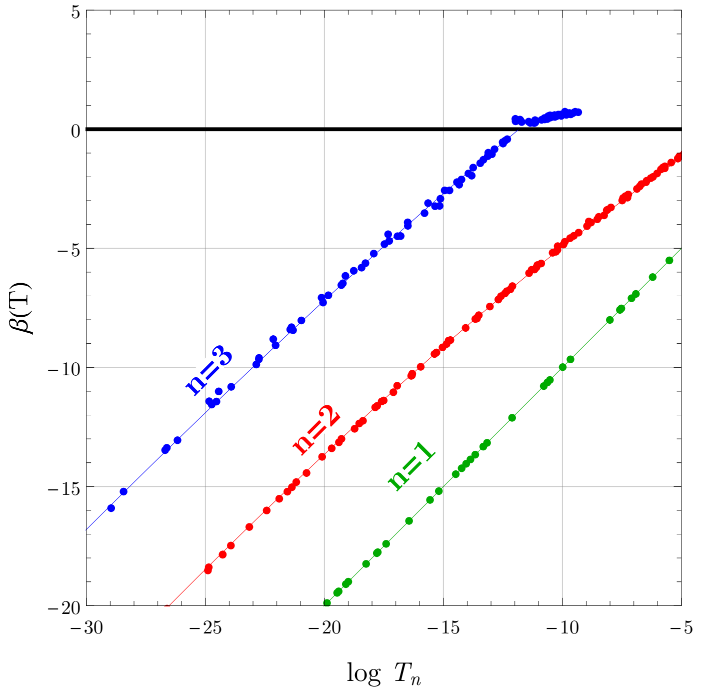
The **scaling function** f_n can be transformed into a **beta function**

$$\log T_n(W,L) = \frac{-2}{f_n(\lambda_n^\infty(W)/L)}.$$

$$\beta(T) = \frac{d\log T_n}{d\log L}$$

$$= \log T_n \frac{d\log f_n(x)}{d\log x}$$

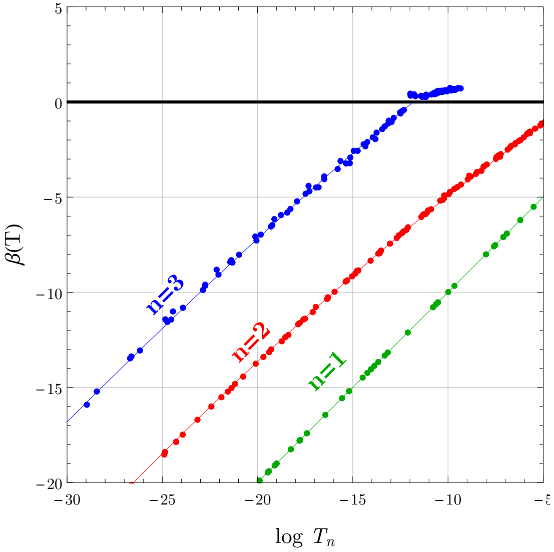
n-particle states in *d*=1



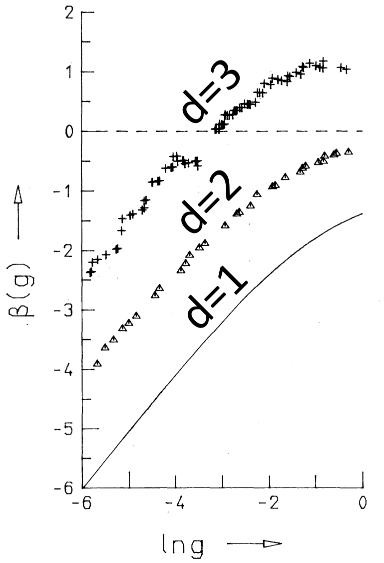
Few-body delocalization

Beta function for:

n-particle states in *d*=1



1-particle states



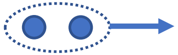
Delocalization transition
for ***n*** particles in ***d*** dimensions when

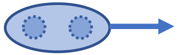
$n + d > 3$

Ref: MacKinnon PRL 1981

Why is this possible?

In **d=2**:

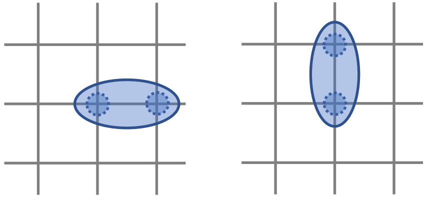

Bound state of two particles


Single particle with **internal structure**

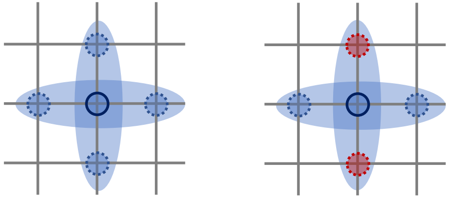
Sigma models with **symplectic symmetry** allow for delocalization in *d=2*

*(**spin-orbit** coupling)*

Example: Spinless fermions with nearest neighbor interaction



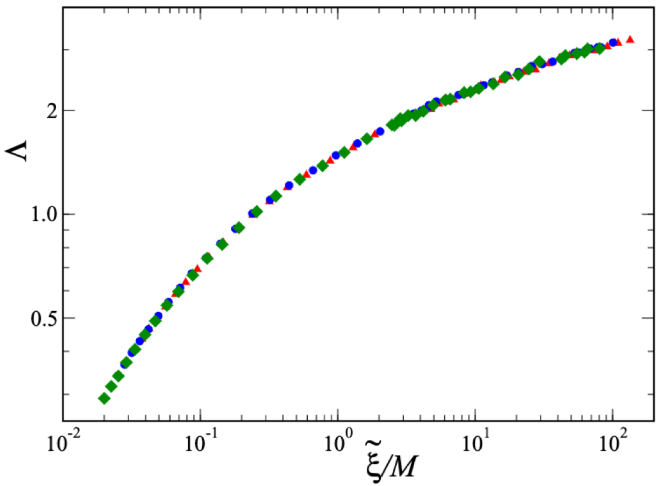
$$|Rx\rangle \qquad |Ry\rangle$$



$$|\psi_{R\pm}\rangle = \frac{1}{2}\left[(|Rx\rangle + |(R-x)x\rangle) \pm (|Ry\rangle + |(R-y)y\rangle)\right]$$

Is it really true?

Criticism in ***d=2***:
"just **finite size effects** in numerics"

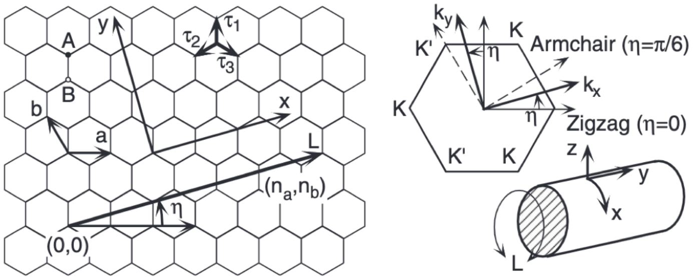


*… but these are results at **weak** disorder*

Ref: Stellin, Orso PRB 2020

Criticism in ***d=1:***

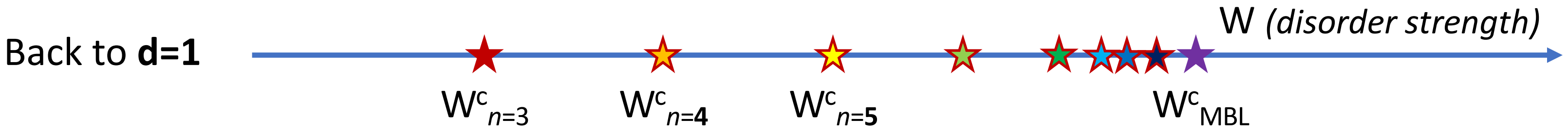
Three-particle states **do not have clear symplectic** symmetry

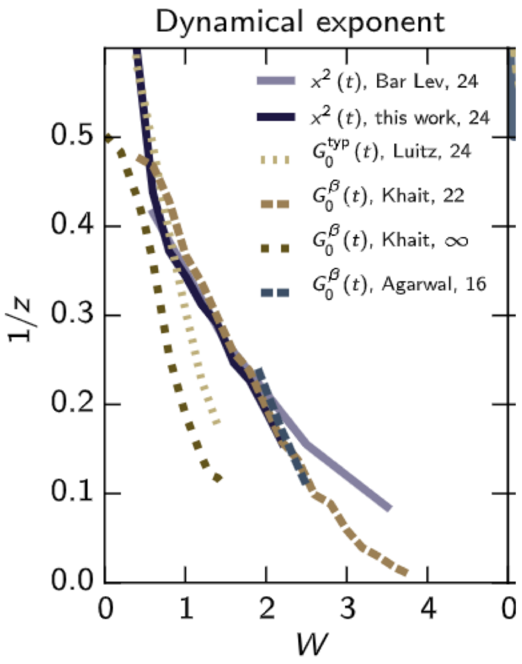


*… but d=1 delocalization **does exist**!*

Ref: Evers, Mirlin RMP 2008

On to **many**-body delocalization





Critical disorder for *n*-body delocalization increases with *n* up to the critical disorder for **many-body delocalization**

Energy/charge transport becomes **increasingly difficult** when $W > W^c_n$ for large *n*

Possible mechanism for **subdiffusion?**

$$x^2(t) \sim t^{2/z}$$

Ref: Luitz, Bar Lev 2017

Acknowledgements

Geneva, CH

Dima Abanin

Murcia, Spain

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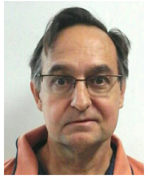
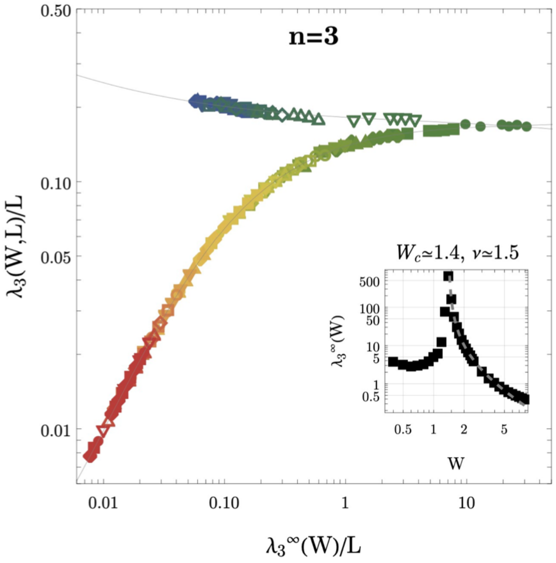


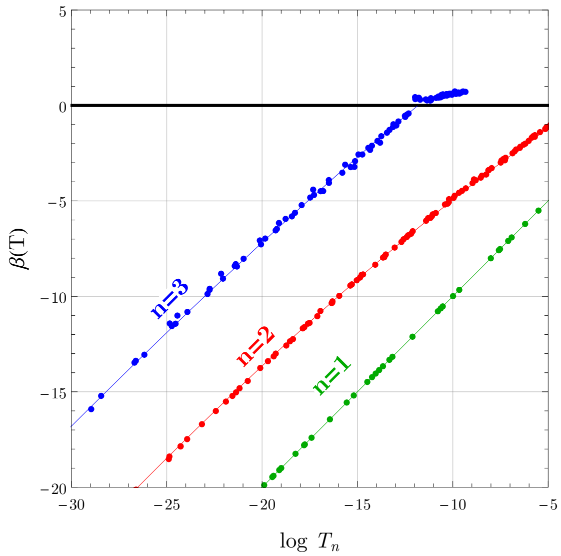
Andres Somoza

Conclusion

Delocalization transition for ***n*** particles in ***d*** dimensions when

n + ***d*** > 3

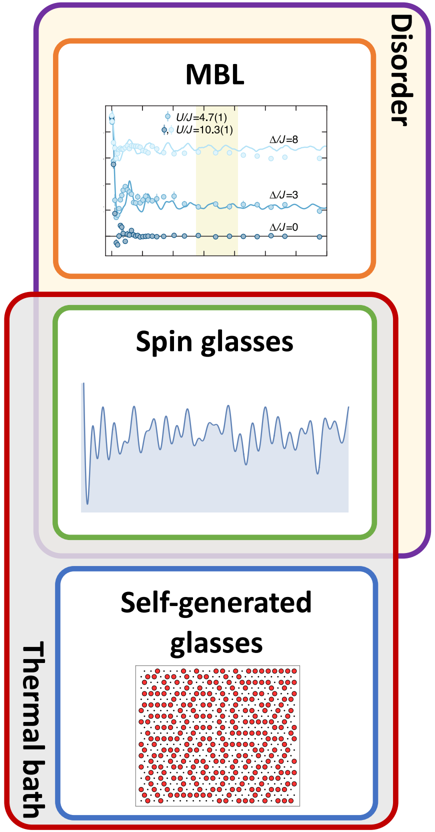


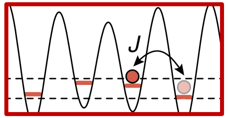


Reference: Rademaker, Phys. Rev. B 104, 214204 (2021)

Extra slides

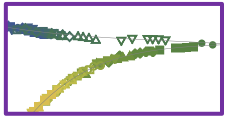
How to break thermalization





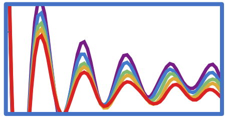
Calculate (Local) **Integrals of Motion**

Ref: Rademaker, Ortuño, PRL 2016



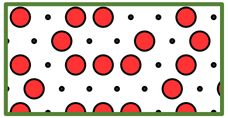
Scaling theory of **few-body delocalization**

Ref: Rademaker, PRB 2021



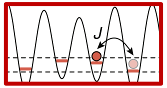
Dynamics of a **quantum spin glass**

Ref: Rademaker, Abanin, PRL 2020



The landscape of a **self-generated electron glass**

Ref: Mahmoudian, Rademaker, et al., PRL 2015



Failure of perturbation theory

$$H = \sum_i \epsilon_i n_i + \frac{1}{2}\sum_{ijkl} V_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

Anderson LIOMs **interactions**

Perturbative construction: dress the electrons with particle-hole excitations

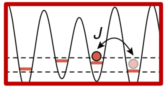
$$c_i \to c_i + \frac{V_{ijkl}}{\epsilon_i + \epsilon_j - \epsilon_k - \epsilon_l} c_j^\dagger c_k \, c_l$$

particle-hole excitation

2nd order **perturbation** theory

This guy can **blow up** due to **resonances**!

Ref: **Rademaker**, Ortuño, PRL 2016; Rademaker, Ortuño, Somoza Ann Phys 2017



Displacement transformations

$$H = \sum_i \epsilon_i n_i + \frac{1}{2} \sum_{ijkl} V_{ijkl} c_i^\dagger c_j^\dagger c_k c_l$$

Anderson LIOMs **interactions**

Our solution: Consider **one** interaction term:

$$X = c_i^\dagger c_j^\dagger c_k c_l$$

$$H = \sum_m \epsilon_m n_m + V_{ijkl}(X + X^\dagger)$$

Introduce **displacement transformations**

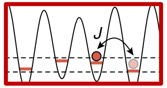
$$\mathcal{D}_\lambda(X) = \exp\left(\lambda(X^\dagger - X)\right)$$

$$\tan 2\lambda = -\frac{V_{ijkl}}{\epsilon_i + \epsilon_j - \epsilon_k - \epsilon_l}$$

The interaction term **disappeared!**

$$\mathcal{D}_\lambda^\dagger(X)\, H\, \mathcal{D}_\lambda(X) = \sum_i \epsilon_i n_i + \sum_{ij} V_{ij} n_i n_j + \ldots$$

Ref: **Rademaker**, Ortuño, PRL 2016; Rademaker, Ortuño, Somoza Ann Phys 2017



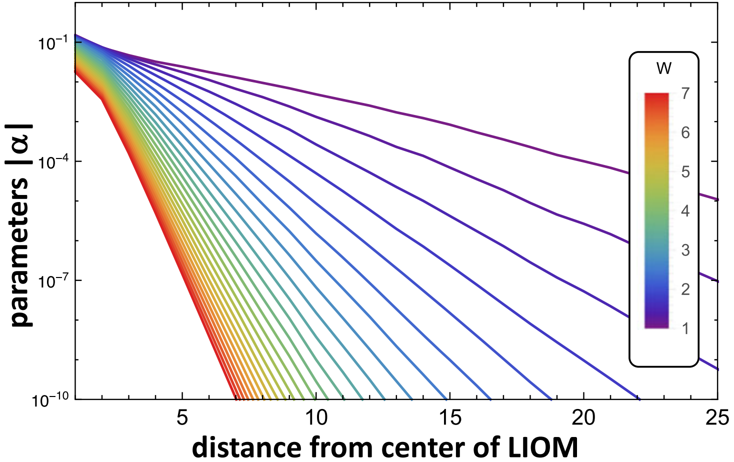
Compute Local Integrals of Motion

Repeated displacement transformations: $U = \mathcal{D}_{\lambda_1}(X_1)\,\mathcal{D}_{\lambda_2}(X_2)\cdots$

Local **integrals of motion**: $\tilde{n}_i = U n_i U^\dagger = n_i + \alpha_{i;jklm} c_j^\dagger c_k^\dagger c_l c_m + \ldots$

Classical Hamiltonian: $H = \sum_i \epsilon_i \tilde{n}_i + \sum_{ij} V_{ij} \tilde{n}_i \tilde{n}_j + \ldots$

approximation: cut-off expansion



Study **1d Anderson** insulator + NN **repulsion**

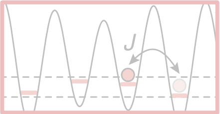
$$H = \sum_{i=1}^{N} \epsilon_i n_i + t \sum_{i=1}^{N-1} (c_i^\dagger c_{i+1} + c_{i+1}^\dagger c_i) + V \sum_{i=1}^{N-1} n_i n_{i+1}$$

Disorder scale: $\epsilon_i \in [-W/2, W/2]$

- **Large** system sizes (*N* up to 60)
- Cannot describe **ergodic transition (?!)**

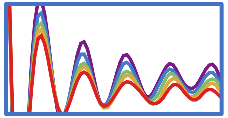
Ref: **Rademaker**, Ortuño, PRL 2016; Rademaker, Ortuño, Somoza Ann Phys 2017

Bring on the bath

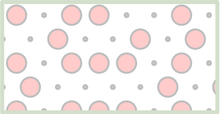


How to calculate (Local) **Integrals of Motion**

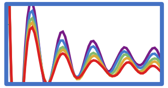
Scaling theory of **few-body delocalization**



Dynamics of a **quantum spin glass**



The landscape of a **self-generated electron glass**



Quantum spin glass

Only known **one-dimensional** spin-glass has **long-range interactions**

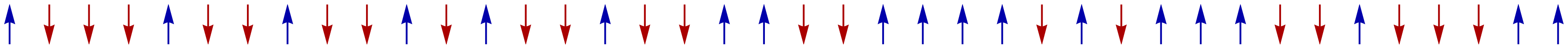
$$H = \sum_{ij} \frac{J_{ij}}{|i-j|^{\alpha}} Z_i Z_j$$

spins Z = ± 1

transverse field creates dynamics

0.5 < α < 1 to have **spin glass order** with random J_{ij}

Kotliar et al, PRB 1983

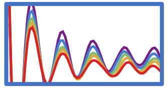


Each spins feels an **effective field** from all the other spins

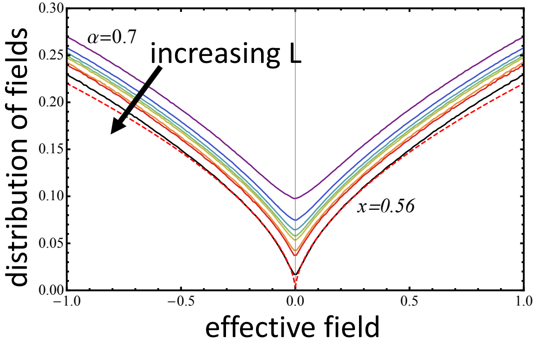
$$\phi_i \equiv \sum_{j} \frac{J_{ij}}{|i-j|^{\alpha}} Z_j$$

With **transverse field**, only **resonant** spins $|\phi_i| < h_x$ will **flip**

Ref: **Rademaker**, Abanin, PRL 2020



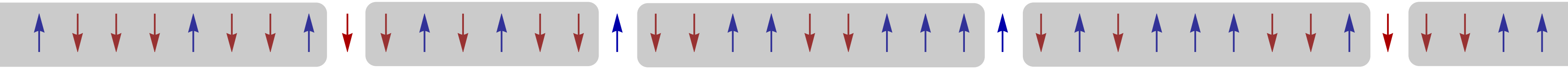
Dynamics of quantum spin glass

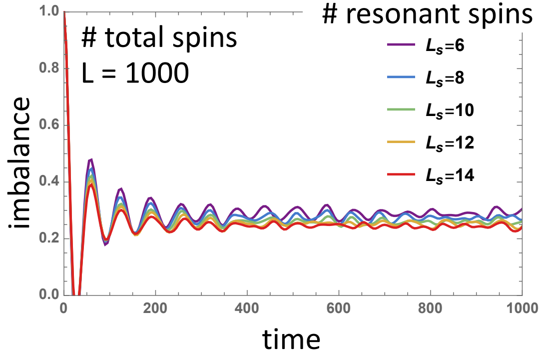


At low T, the distribution of effective fields has a **soft gap**

Resonant spins are **very scarce**

Dynamics: *Most* spins remain frozen,
only resonant spins **entangle**





Numerical technique: **Monte Carlo** to find low T state

+ **Exact Diagonalization** for resonant spins

Verified **new phase**: **ergodic** for resonant spins

localized for other spins

Ref: **Rademaker**, Abanin, PRL 2020

Experimental realization

Hyperfine states of Yb ions allows **exactly the right Hamiltonian**

nature physics — LETTERS

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Many-body localization in a quantum simulator with programmable random disorder

J. Smith[1*], A. Lee[1], P. Richerme[2], B. Neyenhuis[1], P. W. Hess[1], P. Hauke[3,4], M. Heyl[3,4,5], D. A. Huse[6] and C. Monroe[1]

$$H_{\text{Ising}} = \sum_{i<j} J_{i,j}\sigma_i^x\sigma_j^x + \frac{B}{2}\sum_i \sigma_i^z + \sum_i \frac{D_i}{2}\sigma_i^z \qquad (1)$$

tion.
tunable long-range couplin
ally as $J_{i,j} \propto J_{\max}/|i-j|^{\alpha}$ (re
Hz). Here, we tune α betw
of the data $\alpha \approx 1.13$. We d

But their experiment has **10 spins** only
Typical distance between resonant spins is, for *h=0.05J*,

Infinite temperature

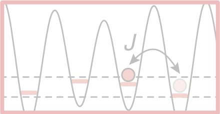
d = 60 sites

Low temperature

d = 100 - 500 sites

Ref: **Rademaker**, Abanin, PRL 2020

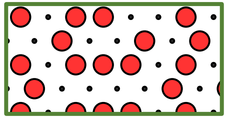
Forget disorder



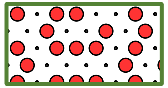
How to calculate (Local) **Integrals of Motion**

Scaling theory of **few-body delocalization**

Dynamics of a **quantum spin glass**



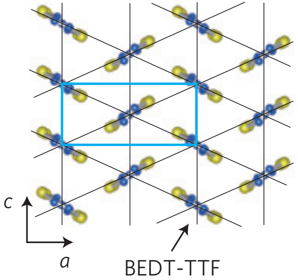
The landscape of a **self-generated electron glass**



Self-generated electron glass

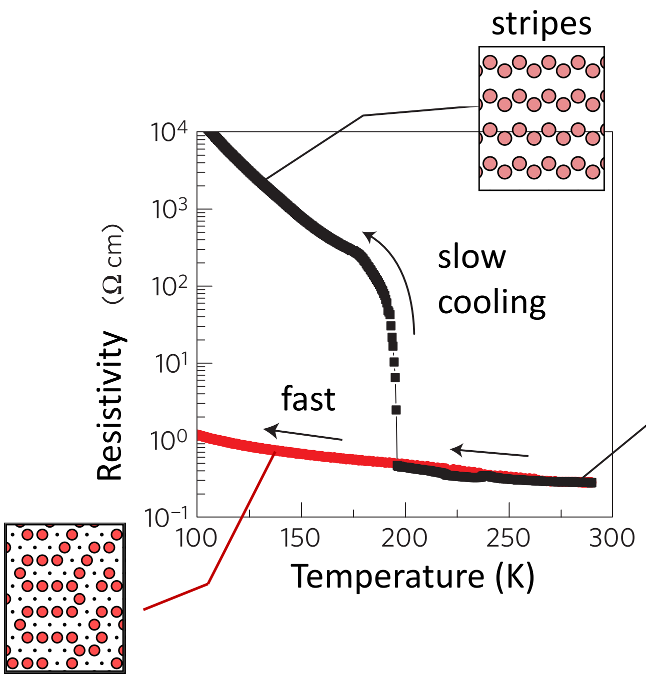
Electrons in organic crystal θ-$(BEDT\text{-}TTF)_2RbZn(SCN)_4$

Ref: Kagawa, Nat Phys 2014; Sato PRB 2014

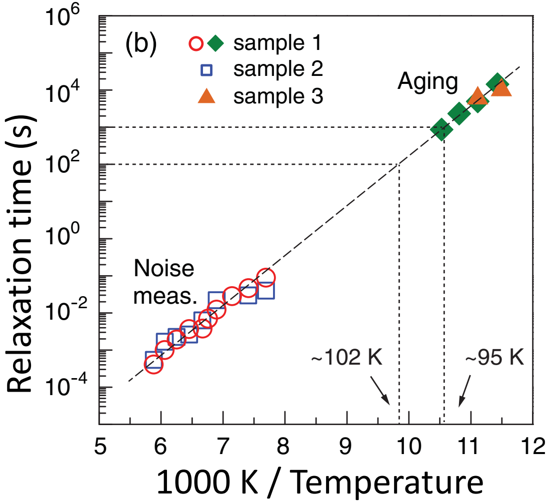


molecules in a **triangular** lattice

one electron per **two** molecules



Arrhenius Law



$$\tau(T) = \tau_0 e^{\Delta/T}$$

Monte Carlo simulations

Electrons on a triangular lattice **with long-range Coulomb interactions**

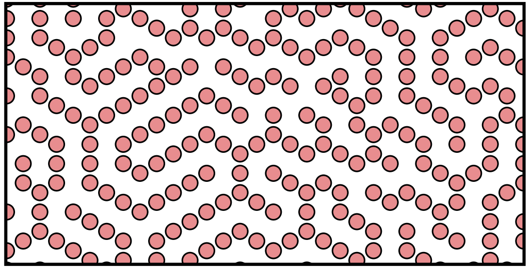
$$H = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + \frac{1}{2}\sum_{ij} V_{ij}\left(n_i - \frac{1}{2}\right)\left(n_j - \frac{1}{2}\right)$$

$$V_{ij} = \frac{V}{|R_i - R_j|}$$

Ground state is **stripe phase**

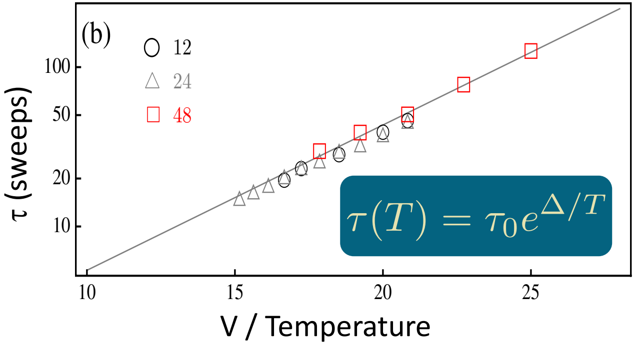
Monte Carlo simulations slow down below at low T

System doesn’t reach stripes but **remains amorphous**

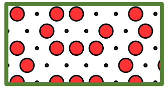


Arrhenius Law

$$C(t+t_w, t_w) = \frac{2}{N}\sum_i \langle \delta n_i(t+t_w)\delta n_i(t_w)\rangle$$



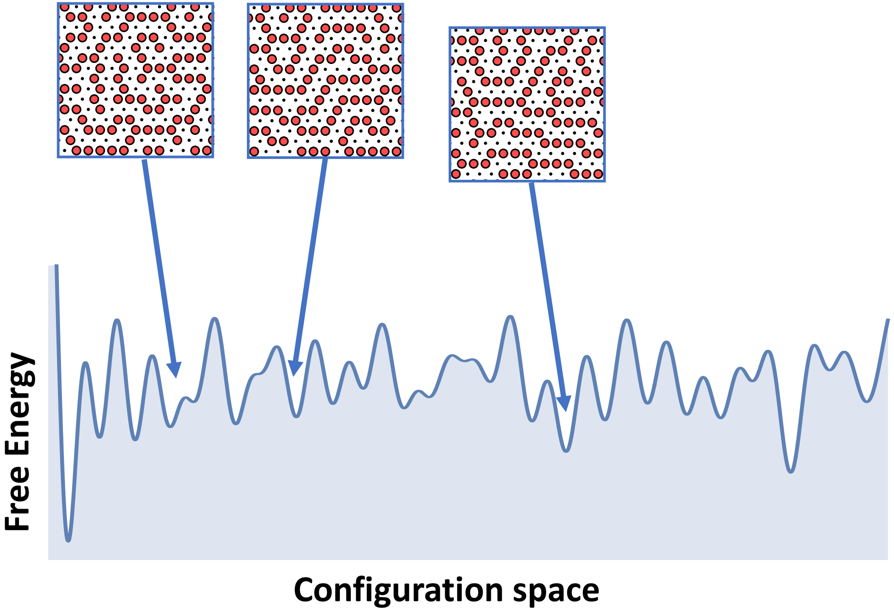
Ref: Mahmoudian, **Rademaker**, et al., PRL 2015; **Rademaker**, et al., NJP 2018



Landscape picture

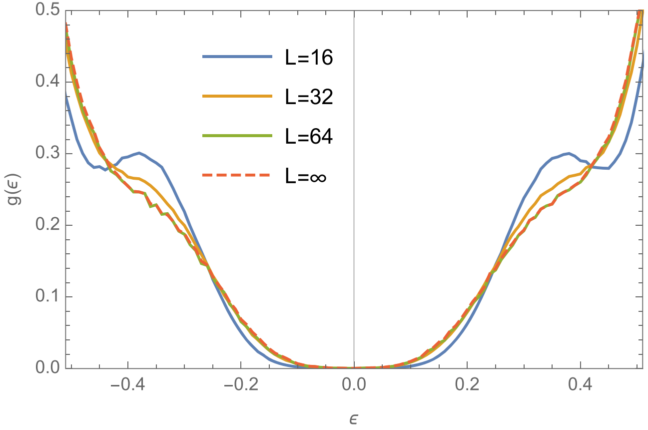
Exponentially many **metastable** states

Example: 24x24 lattice has $\mathbf{10^{35}}$ MS states



DOS satisfies **Efros–Shklovskii bound**

$$g(\epsilon) \leq |\epsilon|^{d-1}$$



without **marginal stability**!

Self-generated glasses are different from **quenched disorder glasses**!

Ref: Mahmoudian, **Rademaker**, et al., PRL 2015; **Rademaker**, et al., NJP 2018