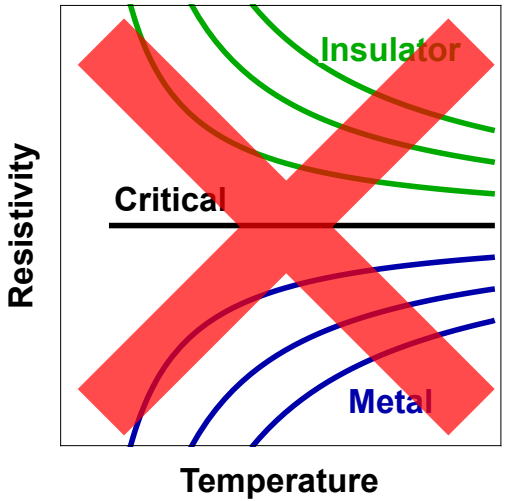
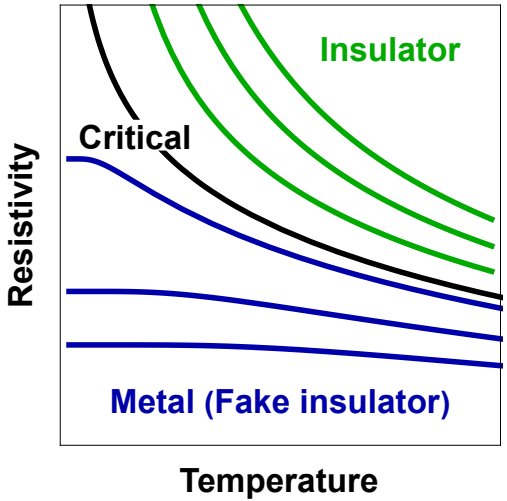
Metal-Insulator Transitions
and the “Fake Insulator” regime



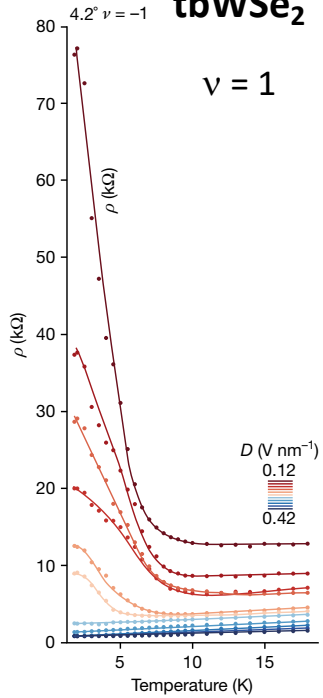


Louk Rademaker, Aspen, 17 March 2023

with Dario Rossi, Johannes Motruk, Dima Abanin (Geneva),
Vlad Dobrosavljevic, Yuting Tan, Henry Tsang (FSU), Simone Fratini (Grenoble)

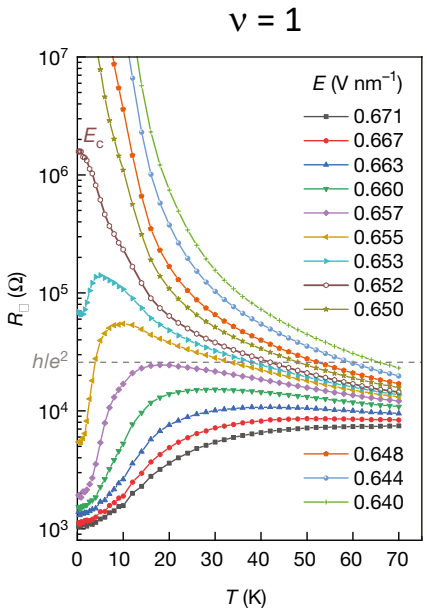
Continuous Quantum Metal-Insulator Transitions (MITs)

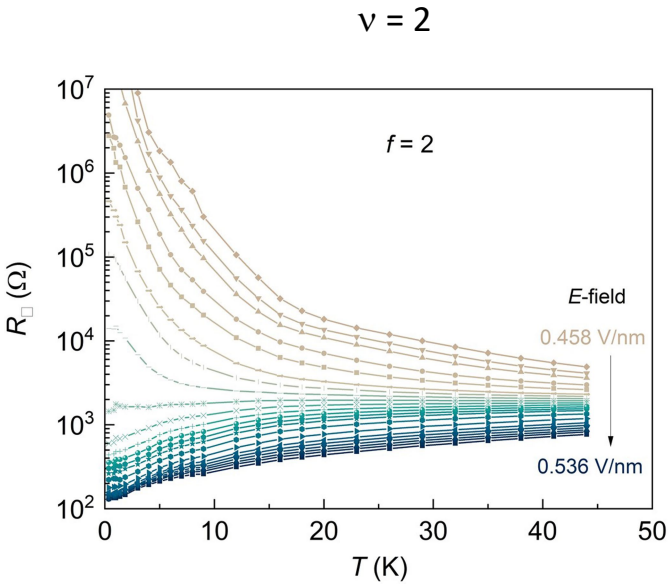
tbWSe$_2$



Ghiotto, …, Dean, Pasupathy Nature 2021

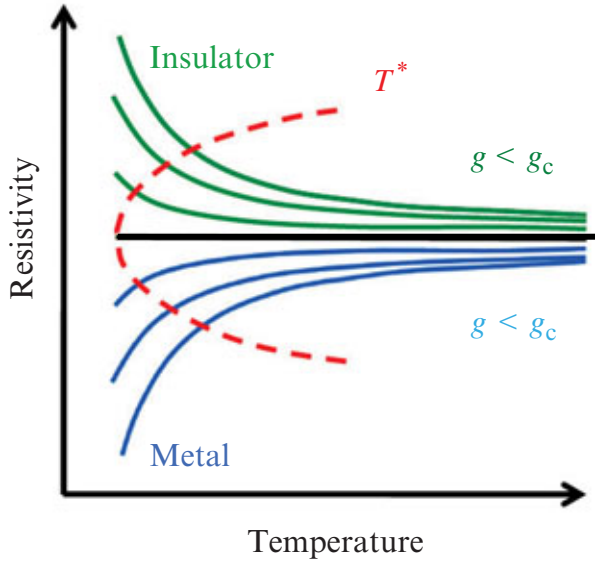
MoTe$_2$/WSe$_2$





Li, …, Shan, Mak, Nature 2021

Traditional theory: Metal vs. Insulator



T=0 **T>0**

$\sigma = 0$ $\rho \sim e^{\Delta/T}$ $d\rho/dT < 0$

activated

Critical: constant $\rho \sim \frac{h}{e^2}$ (25.8 kΩ)

$\sigma > 0$ $\rho \sim \rho_0 + AT^2$ $d\rho/dT > 0$

Fermi liquid

Ref: Dobrosavljevic et al., Conductor-Insulator Quantum Phase Transitions (CUP 2012)

Band theory of MIT: Kubo formula

Kubo formula:
(w/o vertex)

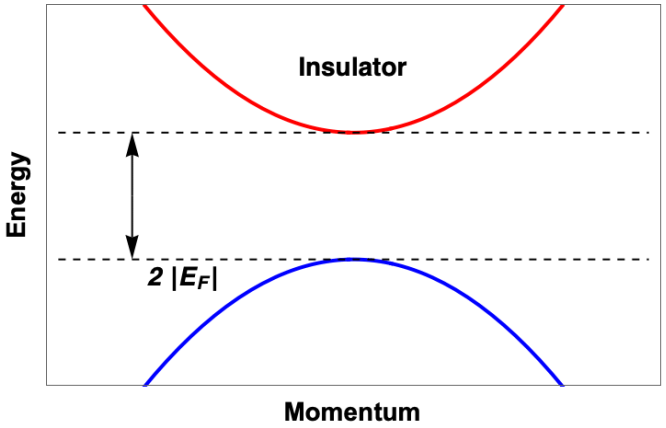
$$\sigma = \frac{\pi}{2} \sum_{\alpha=x,y} \int \frac{d^2p}{(2\pi)^2} \int dz \mathrm{Tr}\left[A(\mathbf{p},z)\mathbf{j}_\alpha(\mathbf{p})A(\mathbf{p},z')\mathbf{j}_\alpha(\mathbf{p})\right]\left(-\frac{\partial n_F(z)}{\partial z}\right)$$

Current operator

Density of states

Fermi function

Apply to **band transition**:



Density of states $A(\mathbf{p},z) = \frac{1}{\pi}\frac{-\mathrm{Im}\Sigma(z)}{(z-\xi_\mathbf{p})^2 + (\mathrm{Im}\Sigma(z))^2}$

with **weak disorder** $-\mathrm{Im}\Sigma(z) = 1/\tau$

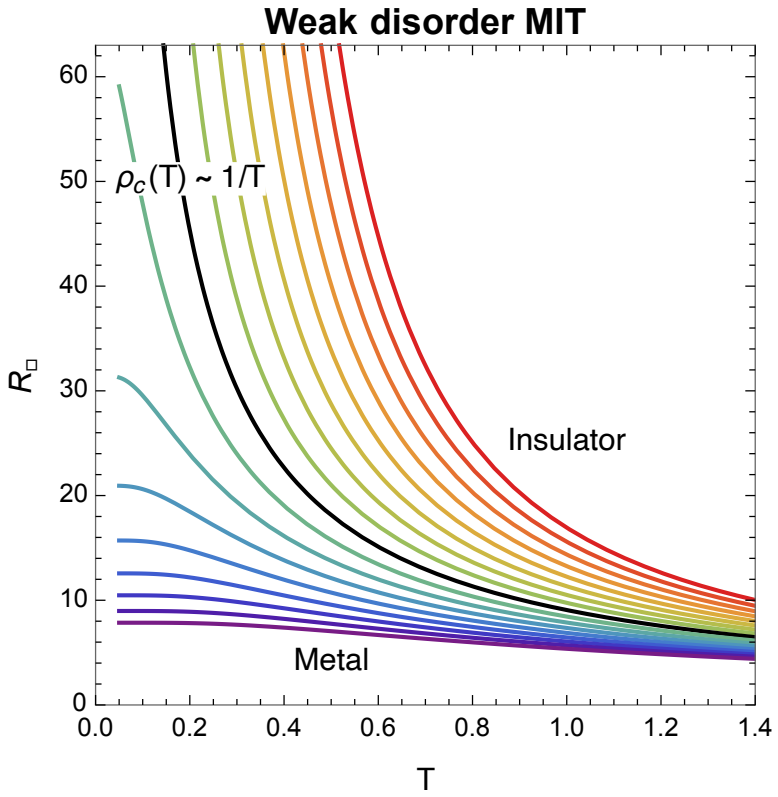
Drude formula $\sigma(T=0) = \frac{e^2 n\tau}{m}$

Particle density

Scattering rate

Effective mass

Band theory of MIT: Exact Solution



$$\sigma(T) = \frac{e^2\tau}{2\pi} T \left(\frac{E_F}{T} + \log\left[1 + e^{-\frac{E_F}{T}}\right] \right)$$

Insulator: activated

$$\rho \sim e^{|E_F|/T}$$

Critical: not constant

$$\rho_c \sim 1/T$$

Metal: negative slope!

$$d\rho/dT < 0$$

*"**fake** insulator"*

Band theory of MIT: Scaling

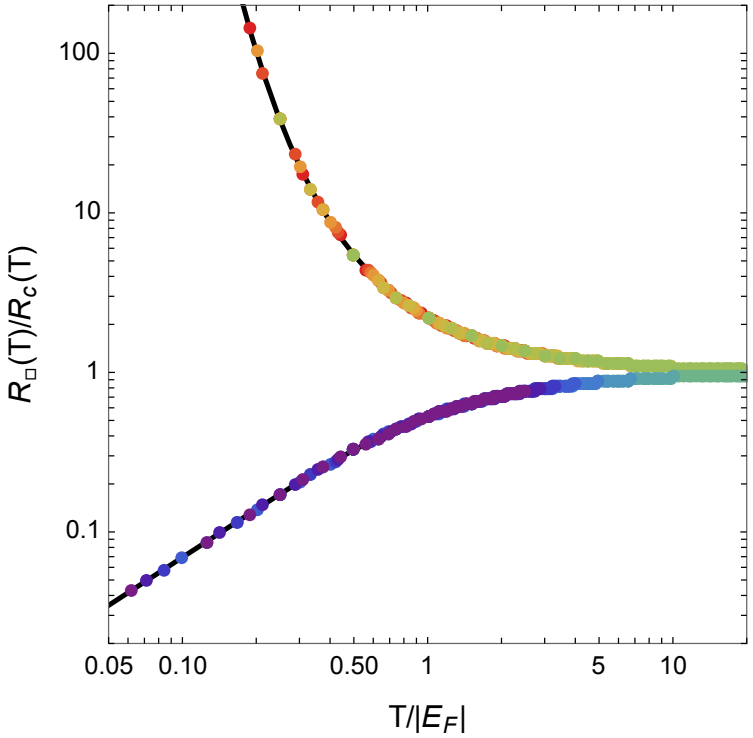
Exact solution

$$\sigma(T) = \frac{e^2\tau}{2\pi} T \left(\frac{E_F}{T} + \log\left[1 + e^{-\frac{E_F}{T}}\right] \right)$$

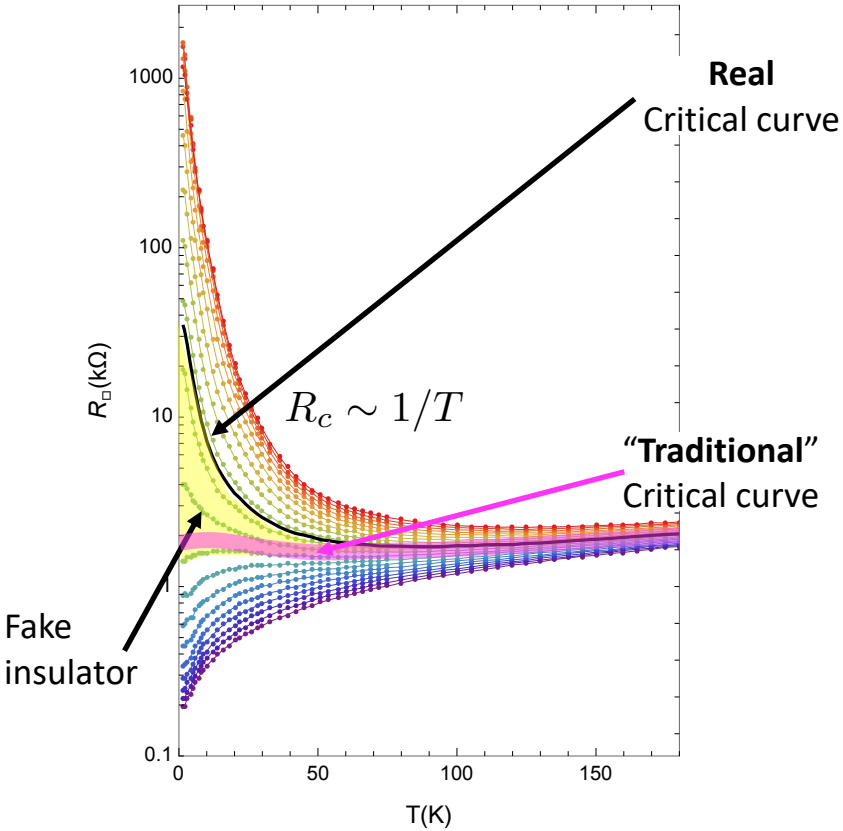
has **scaling form**

$$\sigma(T) = T^{\alpha} f\left(\frac{E_F}{T}\right)$$

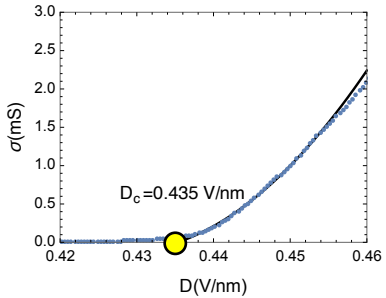
leading to **universal resistivity curves** close to the metal-insulator transition



Application to $MoTe_2/WSe_2$ (ν=2)



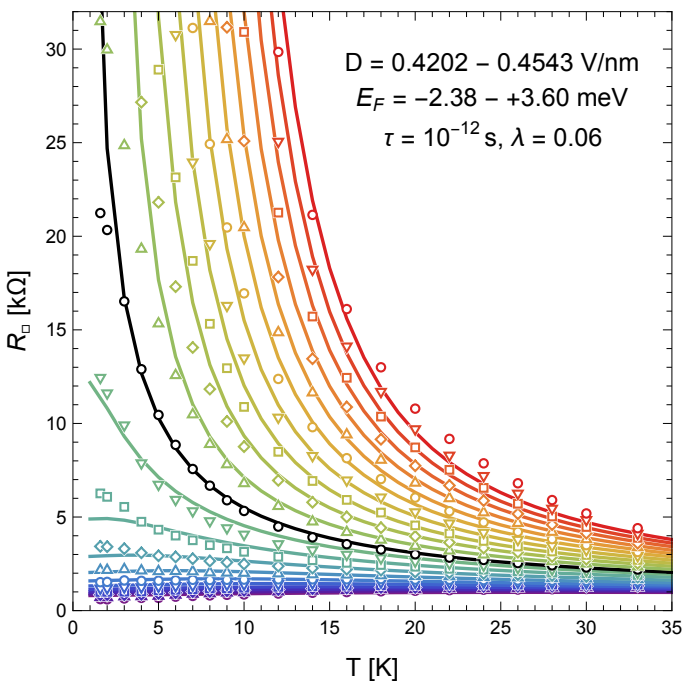
Extrapolating the
zero-temperature conductivity
gives **critical D_c**

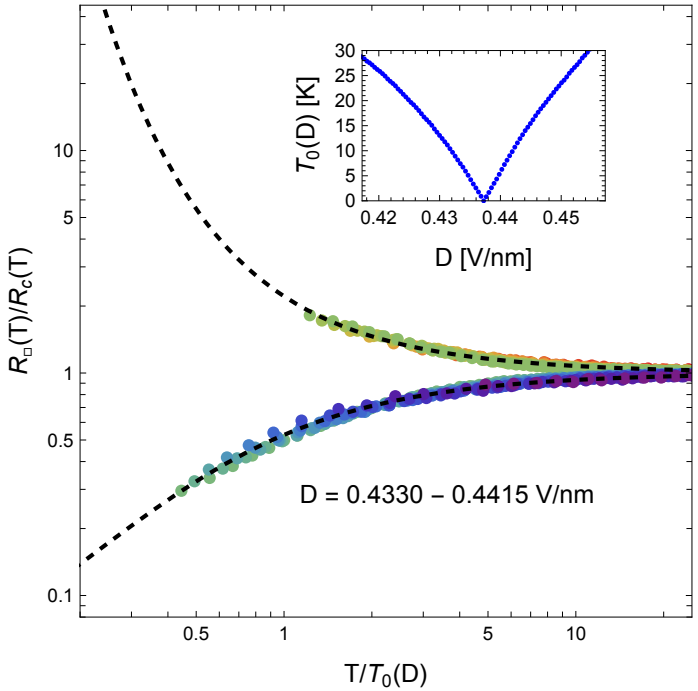


where the resistivity follows

$$R_c \sim 1/T$$

Scaling in $MoTe_2/WSe_2$ (ν=2)





Ref: Li, Nature (2021); Tan, Tsang, Dobrosavljevic (2022)

Mooij correlations as Fake insulators

Mooij: in metals at **high temperature**

$$\alpha = (T/R)dR/dT$$

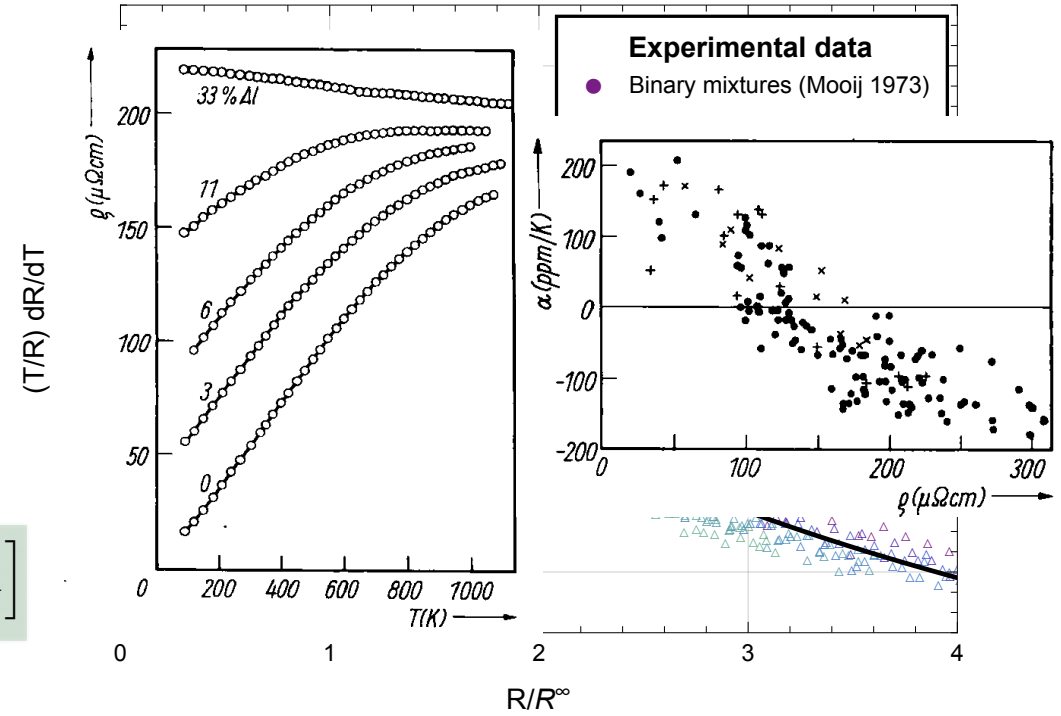
can be **negative** and
is **correlated** with **R**

Assume at high T

$$\tau^{-1} = \tau_0^{-1} + \lambda T$$

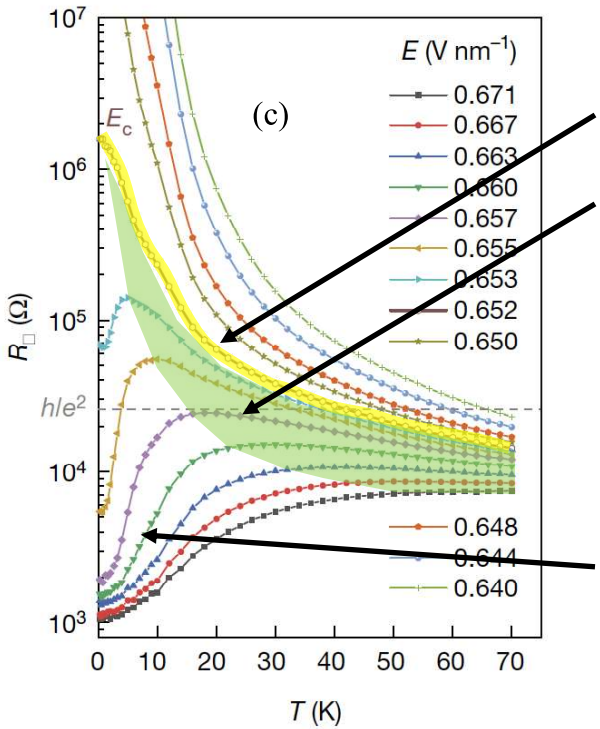
then our **scaling theory** predicts

$$\alpha = \frac{R}{\lambda}\left(1 - 2^{\lambda/R}\right)\log_2\left[2^{\lambda/R} - 1\right]$$



Ref: Mooij (1973); Li, Nature (2021)

Outlook: Mott metal-insulator transition (ν=1)

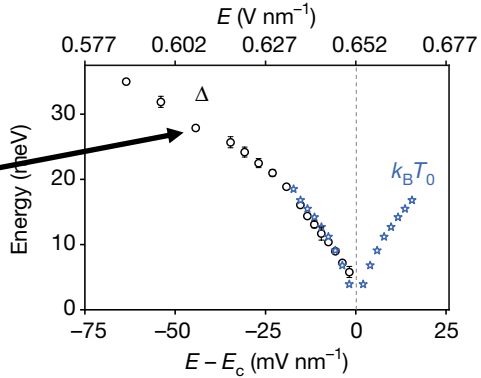


Similarities:

✓ **Powerlaw** critical resistivity curve $R_c(T) \sim T^{-\alpha}$, α = 1.2

✓ "**Fake insulator**" regime

✓ Insulator has continuous **vanishing gap Δ**



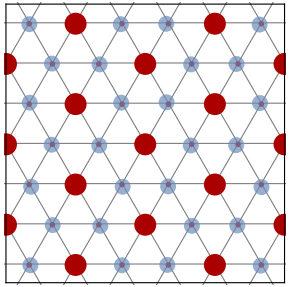
Differences:

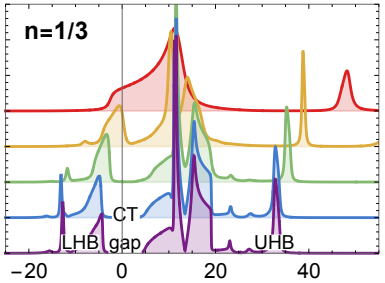
☒ Fermi liquid regime with **mass enhancement**

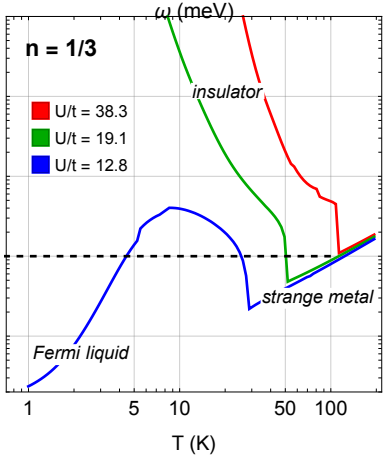
To be continued…

Ref: Tan, Dobrosavljevic, Rademaker, Crystals (2022); Li Nature (2021)

MIT at the Wigner-Mott insulator (ν=1/3 + δ)

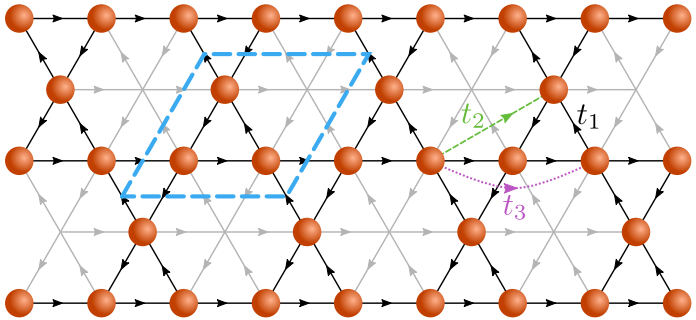


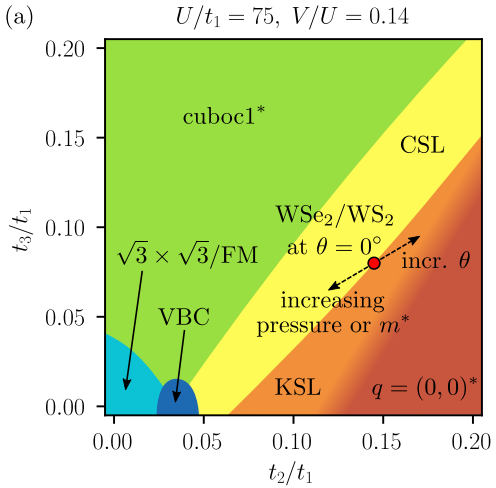


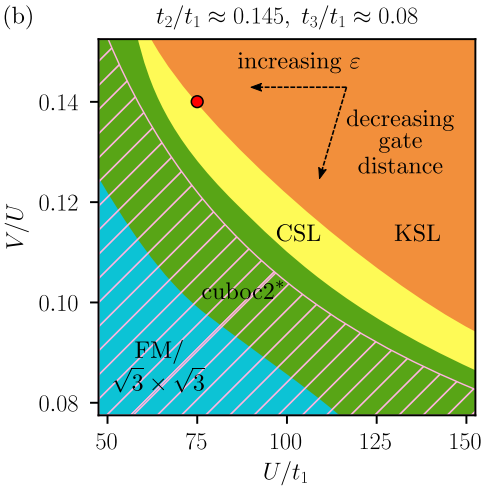


Ref: Tan, Tsang, Dobrosavljevic, Rademaker, arXiv:2210.07926

Kagome and Chiral spin liquid (ν=3/4)







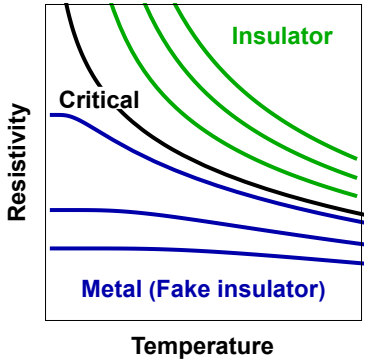
$$H_{\mathrm{spin}} = \sum_{ij} J_{ij} \left[S_i^z S_j^z + \cos\left(\tilde{\phi}_{ij}\right) \left(S_i^x S_j^x + S_i^y S_j^y\right) + \sin\left(\tilde{\phi}_{ij}\right) \left(\mathbf{S}_i \times \mathbf{S}_j\right) \cdot \hat{\mathbf{z}} \right]$$

XXZ **DM**

Ref: Motruk, Rossi, Abanin, Rademaker, arXiv:2211.15696

Conclusions

ν=2



Critical resistivity: **powerlaw diverging**

Metallic regime with $d\rho/dT < 0$ (“**fake**” insulator)

Universal scaling of resistivity curves $\rho(T)/\rho_c(T) = f(T/T_0(\delta))$

Rademaker, Fratini, arXiv:23??.???

ν=1

How to recognize the universal aspects of Mott criticality?

Tan, Dobrosavljevic, Rademaker
arXiv:2206.02055

ν=1/3

Doping a Wigner-Mott insulator

Tan, Tsang, Dobrosavljevic, Rademaker
arXiv:2210.07926

ν=3/4

Kagome Chiral Spin Liquid in TMD heterobilayers

Motruk, Rossi, Abanin, Rademaker
arXiv:2211.15696